Heat and Mass Transfer: Fundamentals & Applications
Fourth Edition
Yunus A. Cengel, Afshin J. Ghajar
McGraw-Hill, 2011

Chapter 11
HEAT EXCHANGERS

TYPES OF HEAT EXCHANGERS

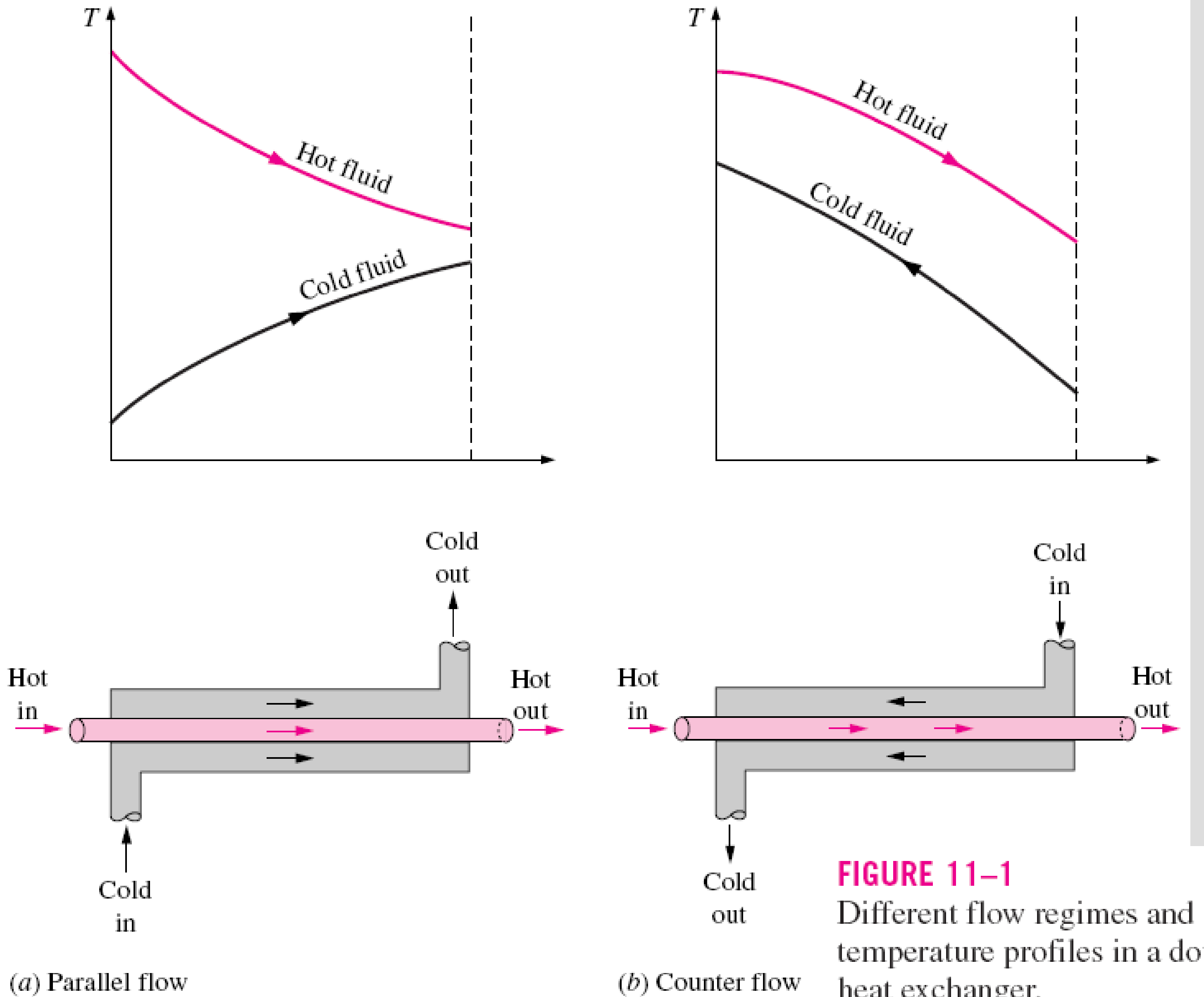


FIGURE 11–1
Different flow regimes and associated temperature profiles in a double-pipe heat exchanger.

Compact heat exchanger: It has a large heat transfer surface area per unit volume (e.g., car radiator, human lung). A heat exchanger with the *area density* $\beta > 700\ m^2/m^3$ is classified as being compact.

Cross-flow: In compact heat exchangers, the two fluids usually move *perpendicular* to each other. The cross-flow is further classified as *unmixed* and *mixed flow.*

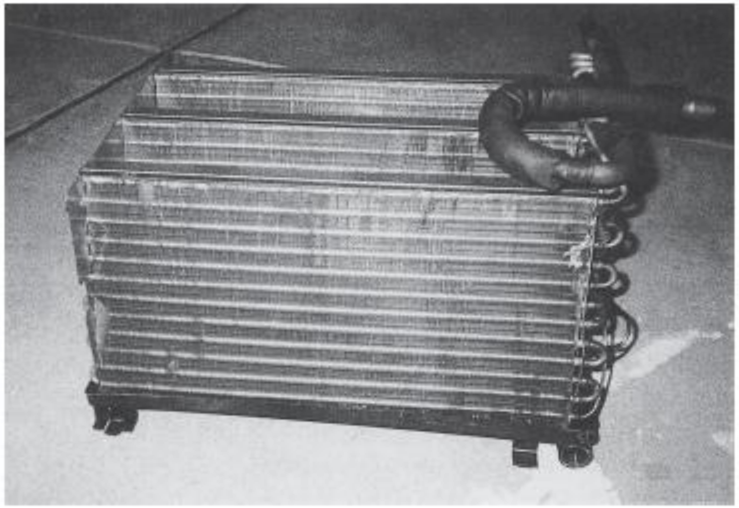


FIGURE 11–2
A gas-to-liquid compact heat exchanger for a residential air-conditioning system.

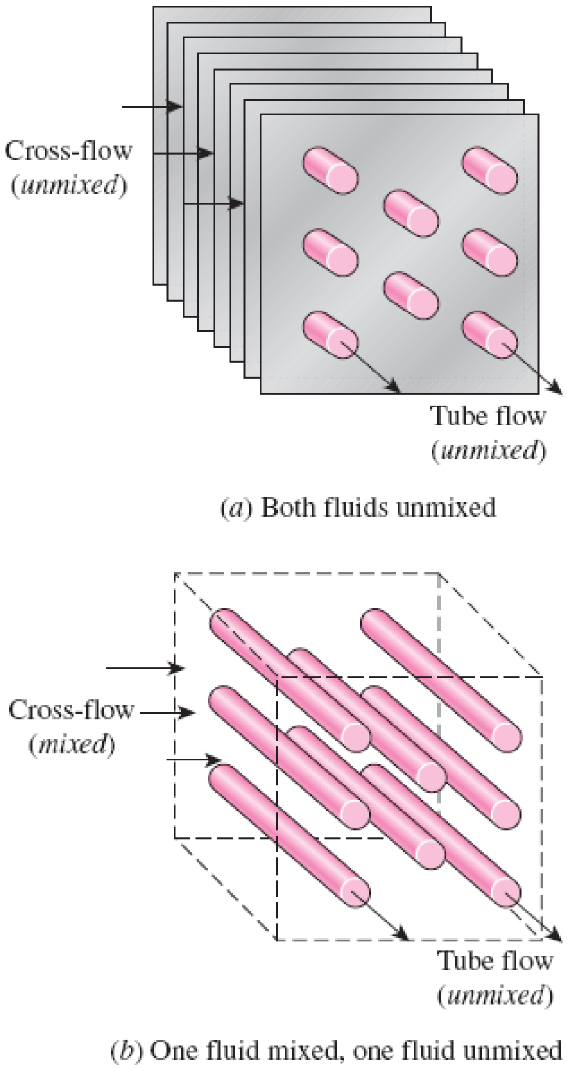


FIGURE 11–3
Different flow configurations in cross-flow heat exchangers.

Shell-and-tube heat exchanger: The most common type of heat exchanger in industrial applications.

They contain a large number of tubes (sometimes several hundred) packed in a shell with their axes parallel to that of the shell. Heat transfer takes place as one fluid flows inside the tubes while the other fluid flows outside the tubes through the shell.

Shell-and-tube heat exchangers are further classified according to the number of shell and tube passes involved.

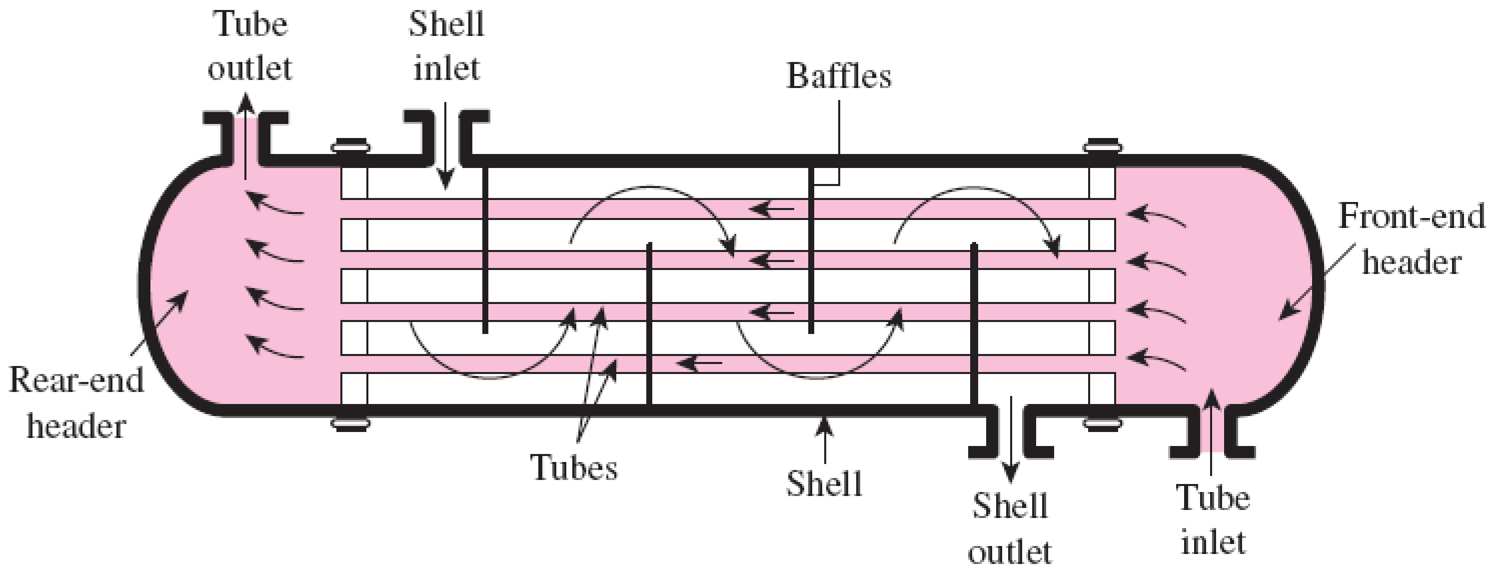


FIGURE 11–4
The schematic of a shell-and-tube heat exchanger (one-shell pass and one-tube pass).

Regenerative heat exchanger: Involves the alternate passage of the hot and cold fluid streams through the same flow area.

Dynamic-type regenerator: Involves a rotating drum and continuous flow of the hot and cold fluid through different portions of the drum so that any portion of the drum passes periodically through the hot stream, storing heat, and then through the cold stream, rejecting this stored heat.

Condenser: One of the fluids is cooled and condenses as it flows through the heat exchanger.

Boiler: One of the fluids absorbs heat and vaporizes.

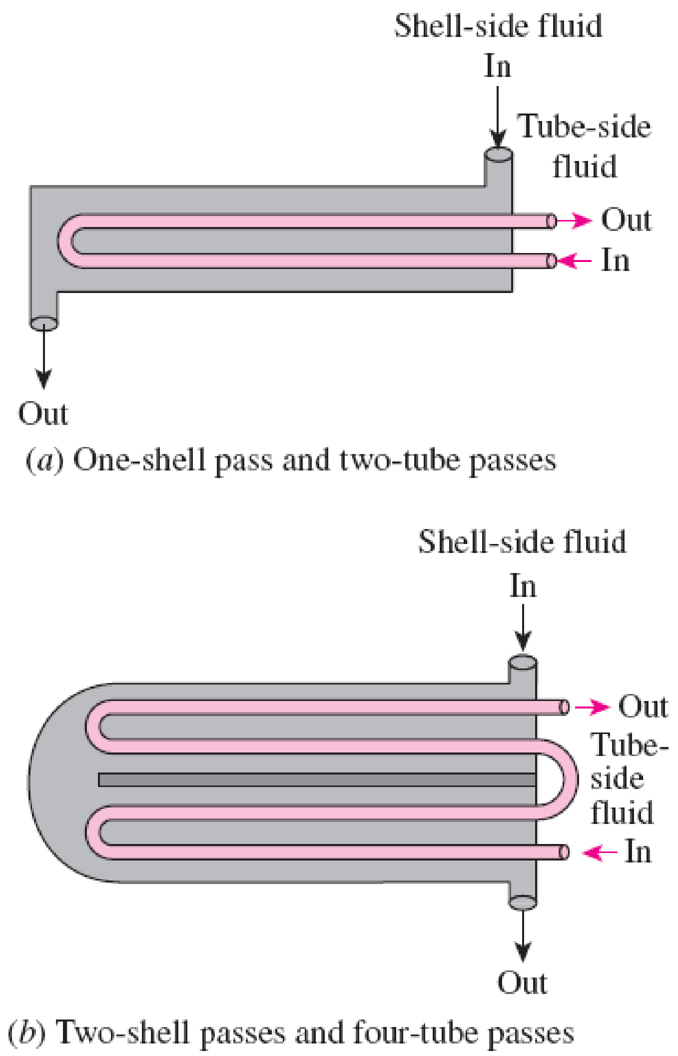
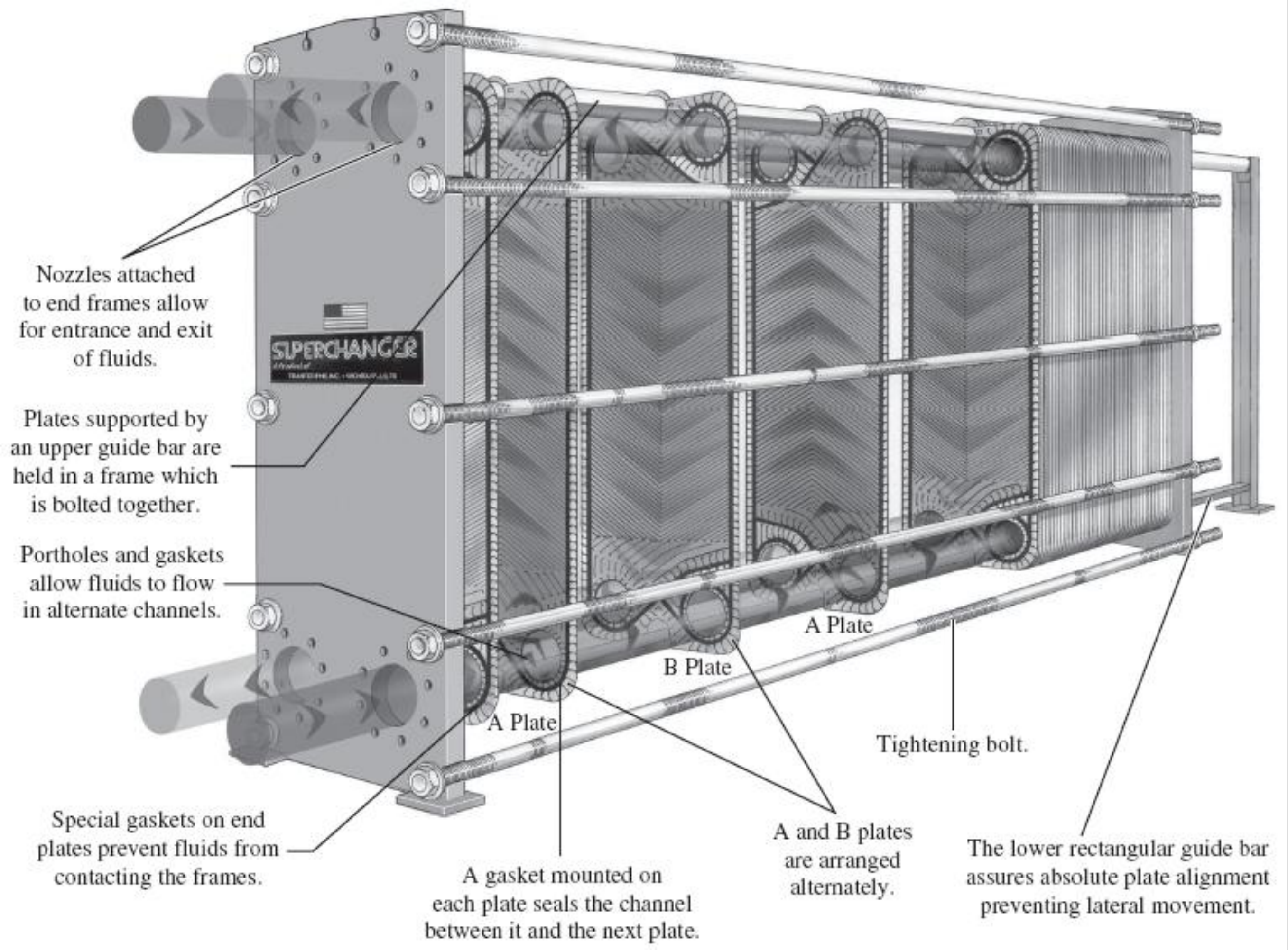


FIGURE 11–5
Multipass flow arrangements in shell-and-tube heat exchangers.

Plate and frame (or just plate) heat exchanger: Consists of a series of plates with corrugated flat flow passages. The hot and cold fluids flow in alternate passages, and thus each cold fluid stream is surrounded by two hot fluid streams, resulting in very effective heat transfer. Well suited for liquid-to-liquid applications.

A plate-and-frame liquid-to-liquid heat exchanger.



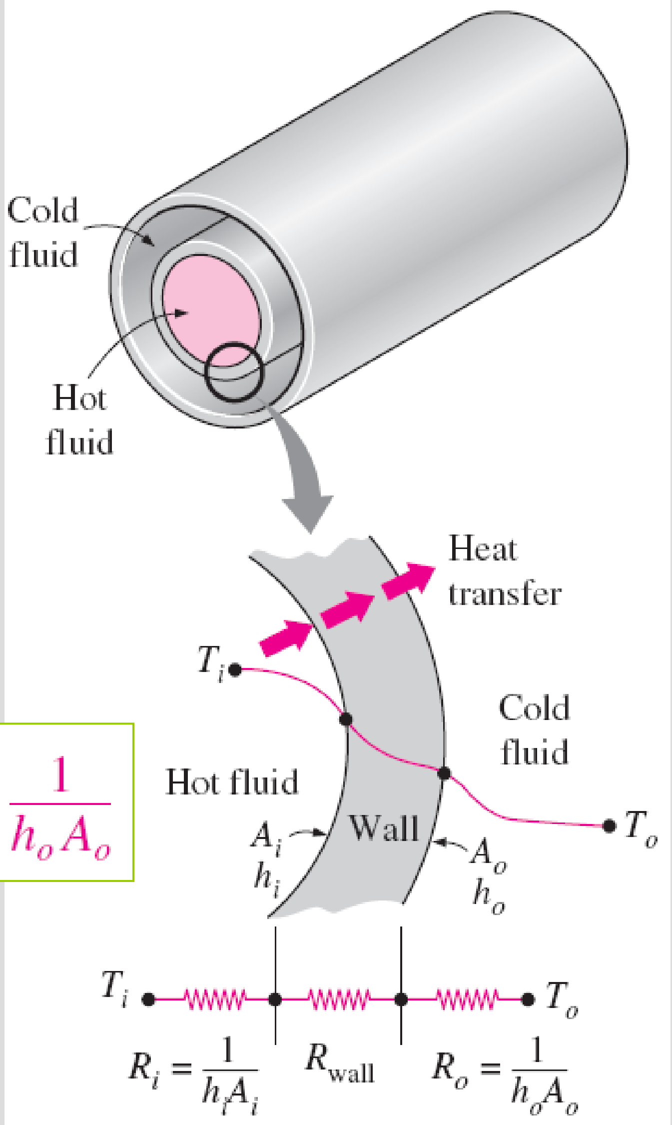
THE OVERALL HEAT TRANSFER COEFFICIENT

- A heat exchanger typically involves two flowing fluids separated by a solid wall.
- Heat is first transferred from the hot fluid to the wall by *convection*, through the wall by *conduction*, and from the wall to the cold fluid again by *convection.*
- Any radiation effects are usually included in the convection heat transfer coefficients.

$$R_{\text{wall}} = \frac{\ln(D_o/D_i)}{2\pi kL}$$

$$R = R_{\text{total}} = R_i + R_{\text{wall}} + R_o = \frac{1}{h_i A_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{1}{h_o A_o}$$

$$A_i = \pi D_i L \text{ and } A_o = \pi D_o L$$



Thermal resistance network associated with heat transfer in a double-pipe heat exchanger.

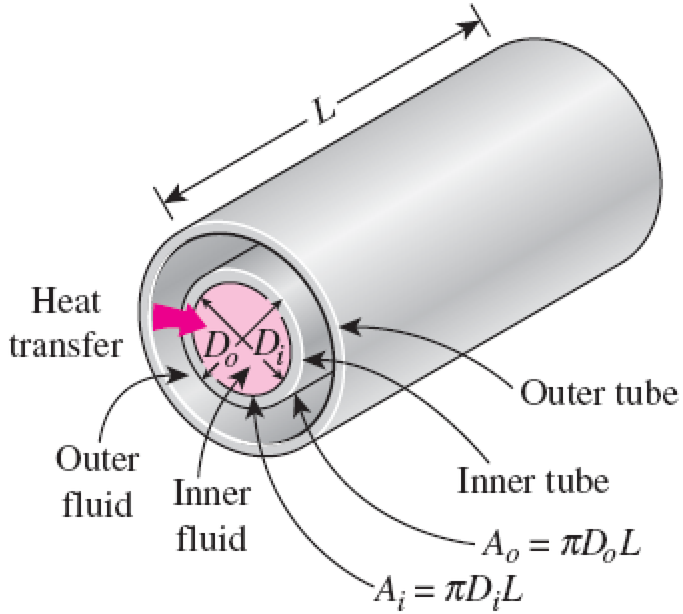


FIGURE 11–8
The two heat transfer surface areas associated with a double-pipe heat exchanger (for thin tubes, $D_i \approx D_o$ and thus $A_i \approx A_o$).

$$\dot{Q} = \frac{\Delta T}{R} = UA\Delta T = U_i A_i \Delta T = U_o A_o \Delta T$$

U the overall heat transfer coefficient, W/m²·°C

$$\frac{1}{UA_s} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o} = R = \frac{1}{h_i A_i} + R_{\text{wall}} + \frac{1}{h_o A_o}$$

$$U_i A_i = U_o A_o, \text{ but } U_i \neq U_o \text{ unless } A_i = A_o$$

When $R_{\text{wall}} \approx 0$ $A_i \approx A_o \approx A_s$

$$\frac{1}{U} \approx \frac{1}{h_i} + \frac{1}{h_o} \qquad U \approx U_i \approx U_o$$

The overall heat transfer coefficient *U* is dominated by the *smaller* convection coefficient. When one of the convection coefficients is *much smaller* than the other (say, $h_i << h_o$), we have $1/h_i >> 1/h_o$, and thus $U \approx h_i$. This situation arises frequently when one of the fluids is a gas and the other is a liquid. In such cases, fins are commonly used on the gas side to enhance the product *UA* and thus the heat transfer on that side.

The overall heat transfer coefficient ranges from about 10 W/m^2·°C for gas-to-gas heat exchangers to about 10,000 W/m^2·°C for heat exchangers that involve phase changes.

When the tube is *finned* on one side to enhance heat transfer, the total heat transfer surface area on the finned side is

$$A_s = A_{\text{total}} = A_{\text{fin}} + A_{\text{unfinned}}$$

For short fins of high thermal conductivity, we can use this total area in the convection resistance relation ***R*_conv = 1/*hA*_s**

$$R_{\text{conv}} = 1/hA_s$$

$$A_s = A_{\text{unfinned}} + \eta_{\text{fin}} A_{\text{fin}}$$

To account for fin efficiency

TABLE 11–1

Representative values of the overall heat transfer coefficients in heat exchangers

Type of heat exchanger	U, W/m^2·K*
Water-to-water	850–1700
Water-to-oil	100–350
Water-to-gasoline or kerosene	300–1000
Feedwater heaters	1000–8500
Steam-to-light fuel oil	200–400
Steam-to-heavy fuel oil	50–200
Steam condenser	1000–6000
Freon condenser (water cooled)	300–1000
Ammonia condenser (water cooled)	800–1400
Alcohol condensers (water cooled)	250–700
Gas-to-gas	10–40
Water-to-air in finned tubes (water in tubes)	30–60†
	400–850†
Steam-to-air in finned tubes (steam in tubes)	30–300†
	400–4000‡

Fouling Factor

The performance of heat exchangers usually deteriorates with time as a result of accumulation of *deposits* on heat transfer surfaces. The layer of deposits represents *additional resistance* to heat transfer. This is represented by a **fouling factor R_f**.

$$\frac{1}{UA_s} = \frac{1}{U_iA_i} = \frac{1}{U_oA_o} = R = \frac{1}{h_iA_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{R_{f,o}}{A_o} + \frac{1}{h_oA_o}$$

The fouling factor increases with the *operating temperature and* the length of service and decreases with the *velocity* of the fluids.

FIGURE 11–9
Precipitation fouling of ash particles on superheater tubes.

TABLE 11–2

Representative fouling factors (thermal resistance due to folling for a unit surface area)

Fluid	R_f, $m^2\cdot K/W$
Distilled water, sea-water, river water, boiler feedwater:	
Below 50°C	0.0001
Above 50°C	0.0002
Fuel oil	0.0009
Steam (oil-free)	0.0001
Refrigerants (liquid)	0.0002
Refrigerants (vapor)	0.0004
Alcohol vapors	0.0001
Air	0.0004

EXAMPLE 13–1 Overall Heat Transfer Coefficient of a Heat Exchanger

Hot oil is to be cooled in a double-tube counter-flow heat exchanger. The copper inner tubes have a diameter of 2 cm and negligible thickness. The inner diameter of the outer tube (the shell) is 3 cm. Water flows through the tube at a rate of 0.5 kg/s, and the oil through the shell at a rate of 0.8 kg/s. Taking the average temperatures of the water and the oil to be 45°C and 80°C, respectively, determine the overall heat transfer coefficient of this heat exchanger.

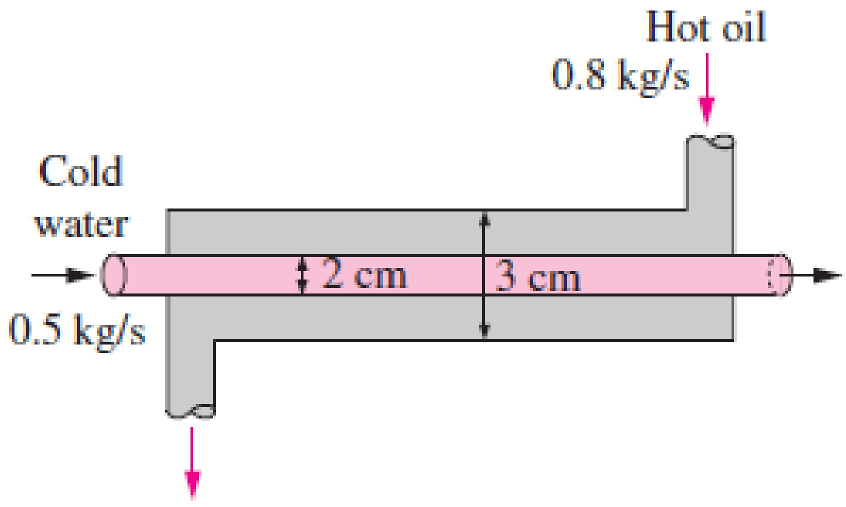


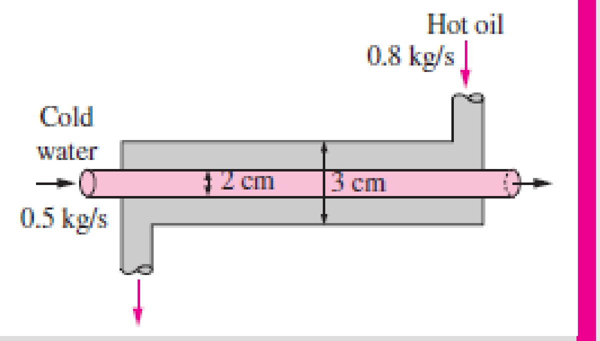
FIGURE 13–10
Schematic for Example 13–1.

SOLUTION Hot oil is cooled by water in a double-tube counter-flow heat exchanger. The overall heat transfer coefficient is to be determined.

Assumptions **1** The thermal resistance of the inner tube is negligible since the tube material is highly conductive and its thickness is negligible. **2** Both the oil and water flow are fully developed. **3** Properties of the oil and water are constant.

Properties The properties of water at 45°C are (Table A–9)

$$\rho = 990 \text{ kg/m}^3 \qquad \text{Pr} = 3.91$$
$$k = 0.637 \text{ W/m}\cdot{}^\circ\text{C} \qquad \nu = \mu/\rho = 0.602 \times 10^{-6} \text{ m}^2\text{/s}$$



The properties of oil at 80°C are (Table A–16).

$$\rho = 852 \text{ kg/m}^3 \qquad \text{Pr} = 490$$
$$k = 0.138 \text{ W/m}\cdot{}^\circ\text{C} \qquad \nu = 37.5 \times 10^{-6} \text{ m}^2\text{/s}$$

Analysis The schematic of the heat exchanger is given in Figure 13–10. The overall heat transfer coefficient U can be determined from Eq. 13-5:

$$\frac{1}{U} \approx \frac{1}{h_i} + \frac{1}{h_o}$$

where h_i and h_o are the convection heat transfer coefficients inside and outside the tube, respectively, which are to be determined using the forced convection relations.

The hydraulic diameter for a circular tube is the diameter of the tube itself, $D_h = D = 0.02$ m. The mean velocity of water in the tube and the Reynolds number are

$$\mathcal{V}_m = \frac{\dot{m}}{\rho A_c} = \frac{\dot{m}}{\rho(\frac{1}{4}\pi D^2)} = \frac{0.5 \text{ kg/s}}{(990 \text{ kg/m}^3)[\frac{1}{4}\pi(0.02 \text{ m})^2]} = 1.61 \text{ m/s}$$

and

$$\text{Re} = \frac{\mathcal{V}_m D_h}{\nu} = \frac{(1.61 \text{ m/s})(0.02 \text{ m})}{0.602 \times 10^{-6} \text{ m}^2\text{/s}} = 53{,}490$$

which is greater than 4000. Therefore, the flow of water is turbulent. Assuming the flow to be fully developed, the Nusselt number can be determined from

$$\text{Nu} = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\text{Pr}^{0.4} = 0.023(53{,}490)^{0.8}(3.91)^{0.4} = 240.6$$

Then,

$$h = \frac{k}{D_h}\text{Nu} = \frac{0.637\ \text{W/m}\cdot{}^\circ\text{C}}{0.02\ \text{m}}(240.6) = 7663\ \text{W/m}^2\cdot{}^\circ\text{C}$$

Now we repeat the analysis above for oil. The properties of oil at 80°C are

$$\rho = 852\ \text{kg/m}^3 \qquad \nu = 37.5\times10^{-6}\ \text{m}^2\text{/s}$$
$$k = 0.138\ \text{W/m}\cdot{}^\circ\text{C} \qquad \text{Pr} = 490$$

The hydraulic diameter for the annular space is

$$D_h = D_o - D_i = 0.03 - 0.02 = 0.01\ \text{m}$$

The mean velocity and the Reynolds number in this case are

$$\mathcal{V}_m = \frac{\dot m}{\rho A_c} = \frac{\dot m}{\rho[\frac{1}{4}\pi(D_o^2 - D_i^2)]} = \frac{0.8\ \text{kg/s}}{(852\ \text{kg/m}^3)[\frac{1}{4}\pi(0.03^2 - 0.02^2)]\ \text{m}^2} = 2.39\ \text{m/s}$$

and

$$\text{Re} = \frac{\mathcal{V}_m D_h}{\nu} = \frac{(2.39\ \text{m/s})(0.01\ \text{m})}{37.5\times10^{-6}\ \text{m}^2\text{/s}} = 637$$

which is less than 4000. Therefore, the flow of oil is laminar. Assuming fully developed flow, the Nusselt number on the tube side of the annular space Nu_i corresponding to $D_i/D_o = 0.02/0.03 = 0.667$ can be determined from Table 13–3 by interpolation to be

and

$$h_o = \frac{k}{D_h}\text{Nu} = \frac{0.138\ \text{W/m}\cdot{}^\circ\text{C}}{0.01\ \text{m}}(5.45) = 75.2\ \text{W/m}^2\cdot{}^\circ\text{C}$$

Then the overall heat transfer coefficient for this heat exchanger becomes

$$U = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}} = \frac{1}{\frac{1}{7663\ \text{W/m}^2\cdot{}^\circ\text{C}} + \frac{1}{75.2\ \text{W/m}^2\cdot{}^\circ\text{C}}} = 74.5\ \text{W/m}^2\cdot{}^\circ\text{C}$$

EXAMPLE 13–2 Effect of Fouling on the Overall Heat Transfer Coefficient

A double-pipe (shell-and-tube) heat exchanger is constructed of a stainless steel (k = 15.1 W/m · °C) inner tube of inner diameter D_i = 1.5 cm and outer diameter D_o = 1.9 cm and an outer shell of inner diameter 3.2 cm. The convection heat transfer coefficient is given to be h_i = 800 W/m^2 · °C on the inner surface of the tube and h_o = 1200 W/m^2 · °C on the outer surface. For a fouling factor of $R_{f,i}$ = 0.0004 m^2 · °C/W on the tube side and $R_{f,o}$ = 0.0001 m^2 · °C/W on the shell side, determine (*a*) the thermal resistance of the heat exchanger per unit length and (*b*) the overall heat transfer coefficients, U_i and U_o based on the inner and outer surface areas of the tube, respectively.

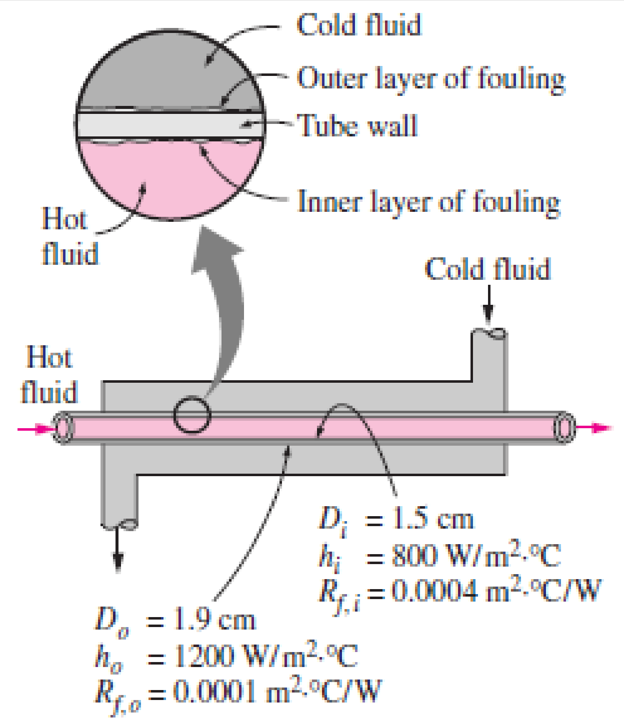


FIGURE 13–11
Schematic for Example 13–2.

SOLUTION The heat transfer coefficients and the fouling factors on the tube and shell sides of a heat exchanger are given. The thermal resistance and the overall heat transfer coefficients based on the inner and outer areas are to be determined.

Assumptions The heat transfer coefficients and the fouling factors are constant and uniform.

Analysis (*a*) The schematic of the heat exchanger is given in Figure 13–11. The thermal resistance for an unfinned shell-and-tube heat exchanger with fouling on both heat transfer surfaces is given by Eq. 13-8 as

$$R = \frac{1}{UA_s} = \frac{1}{U_iA_i} = \frac{1}{U_oA_o} = \frac{1}{h_iA_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{R_{f,o}}{A_o} + \frac{1}{h_oA_o}$$

where

$$A_i = \pi D_i L = \pi(0.015 \text{ m})(1 \text{ m}) = 0.0471 \text{ m}^2$$
$$A_o = \pi D_o L = \pi(0.019 \text{ m})(1 \text{ m}) = 0.0597 \text{ m}^2$$

Substituting, the total thermal resistance is determined to be

$$\begin{aligned} R &= \frac{1}{(800 \text{ W/m}^2\cdot{}^\circ\text{C})(0.0471 \text{ m}^2)} + \frac{0.0004 \text{ m}^2\cdot{}^\circ\text{C/W}}{0.0471 \text{ m}^2} \\ &\quad + \frac{\ln(0.019/0.015)}{2\pi(15.1 \text{ W/m}\cdot{}^\circ\text{C})(1 \text{ m})} \\ &\quad + \frac{0.0001 \text{ m}^2\cdot{}^\circ\text{C/W}}{0.0597 \text{ m}^2} + \frac{1}{(1200 \text{ W/m}^2\cdot{}^\circ\text{C})(0.0597 \text{ m}^2)} \\ &= (0.02654 + 0.00849 + 0.0025 + 0.00168 + 0.01396)^\circ\text{C/W} \\ &= \mathbf{0.0532^\circ C/W} \end{aligned}$$

Note that about 19 percent of the total thermal resistance in this case is due to fouling and about 5 percent of it is due to the steel tube separating the two fluids. The rest (76 percent) is due to the convection resistances on the two sides of the inner tube.

(*b*) Knowing the total thermal resistance and the heat transfer surface areas, the overall heat transfer coefficient based on the inner and outer surfaces of the tube are determined again from Eq. 13-8 to be

$$U_i = \frac{1}{RA_i} = \frac{1}{(0.0532\ °\text{C/W})(0.0471\ \text{m}^2)} = 399\ \text{W/m}^2 \cdot °\text{C}$$

and

$$U_o = \frac{1}{RA_o} = \frac{1}{(0.0532\ °\text{C/W})(0.0597\ \text{m}^2)} = 315\ \text{W/m}^2 \cdot °\text{C}$$

ANALYSIS OF HEAT EXCHANGERS

An engineer often finds himself or herself in a position

1. to *select a heat exchanger* that will achieve a *specified temperature change* in a fluid stream of known mass flow rate - ***the log mean temperature difference* (or LMTD) method**.
2. to *predict the outlet temperatures* of the hot and cold fluid streams in a *specified heat exchanger* - **the *effectiveness–NTU* method**.

The rate of heat transfer in heat exchanger (HE is insulated):

$$\dot{Q} = \dot{m}_c c_{pc}(T_{c,\,out} - T_{c,\,in})$$

$$\dot{Q} = \dot{m}_h c_{ph}(T_{h,\,in} - T_{h,\,out})$$

$\dot{m}_c, \dot{m}_h$ = mass flow rates

c_{pc}, c_{ph} = specific heats

$T_{c,\,out}, T_{h,\,out}$ = outlet temperatures

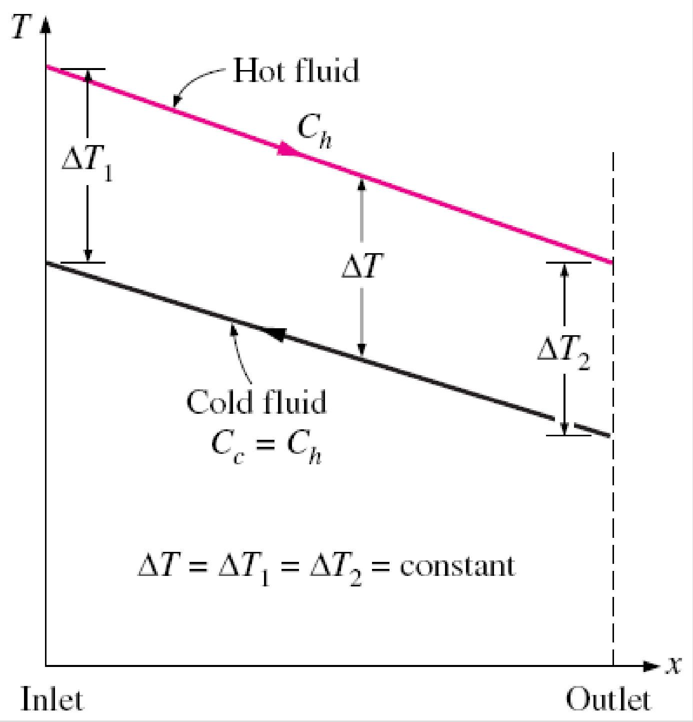
$T_{c,\,in}, T_{h,\,in}$ = inlet temperatures

$$C_h = \dot{m}_h c_{ph} \qquad \text{and} \qquad C_c = \dot{m}_c c_{pc}$$

heat capacity rate

$$\dot{Q} = C_c(T_{c,\,out} - T_{c,\,in}) \qquad \dot{Q} = C_h(T_{h,\,in} - T_{h,\,out})$$

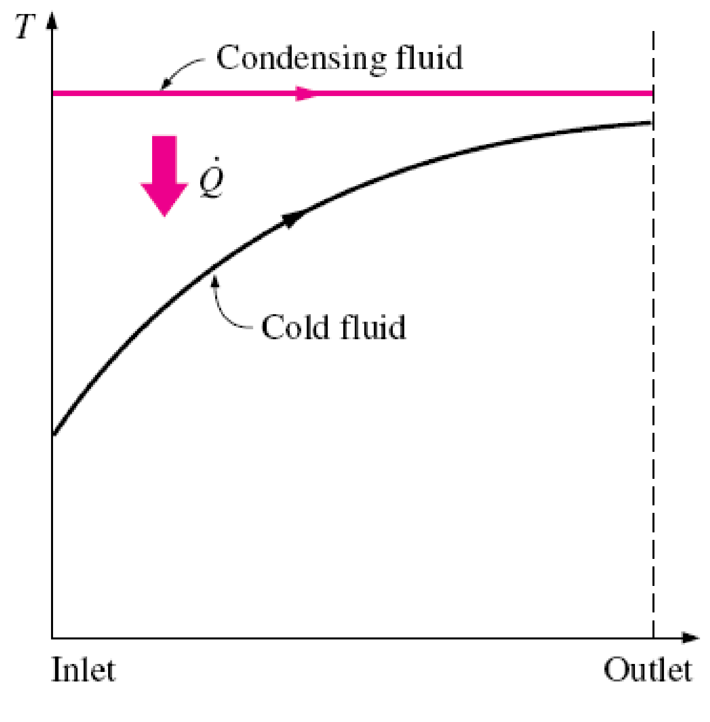
Two fluid streams that have the same capacity rates experience the same temperature change in a well-insulated heat exchanger.



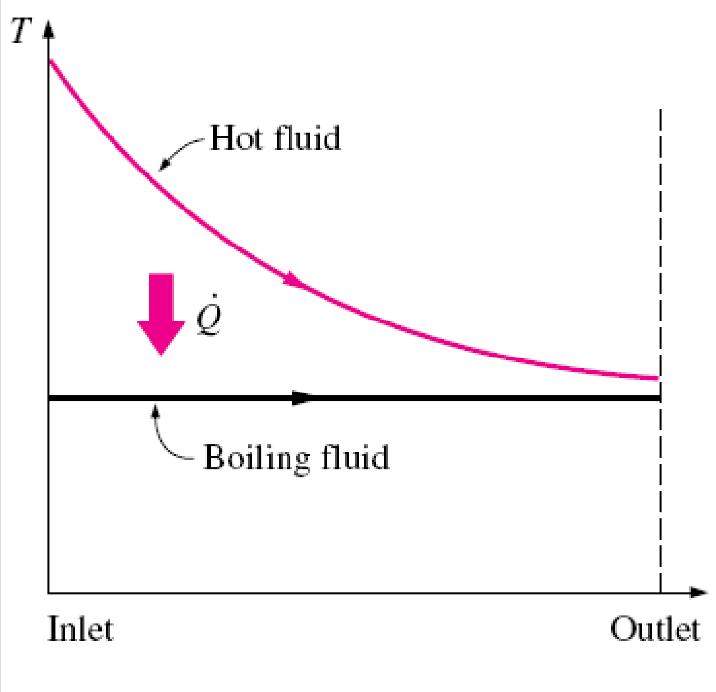
$\dot{m}$ is the rate of evaporation or condensation of the fluid $\dot{Q} = \dot{m}h_{fg}$

h_{fg} is the enthalpy of vaporization of the fluid at the specified temperature or pressure.

The heat capacity rate of a fluid during a phase-change process must approach infinity since the temperature change is practically zero.



(a) Condenser ($C_h \to \infty$)



(b) Boiler ($C_c \to \infty$)

Variation of fluid temperatures in a heat exchanger when one of the fluids condenses or boils.

$\dot{Q} = UA_s\,\Delta T_m$ ΔT_m an appropriate mean (average) temperature difference between the two fluids

THE LOG MEAN TEMPERATURE DIFFERENCE METHOD

$$\delta \dot{Q} = -\dot{m}_h c_{ph}\, dT_h \qquad \delta \dot{Q} = \dot{m}_c c_{pc}\, dT_c$$

$$dT_h = -\frac{\delta \dot{Q}}{\dot{m}_h c_{ph}} \qquad dT_c = \frac{\delta \dot{Q}}{\dot{m}_c c_{pc}}$$

$$dT_h - dT_c = d(T_h - T_c) = -\delta \dot{Q}\left(\frac{1}{\dot{m}_h c_{ph}} + \frac{1}{\dot{m}_c c_{pc}}\right)$$

$$\delta \dot{Q} = U(T_h - T_c)\, dA_s$$

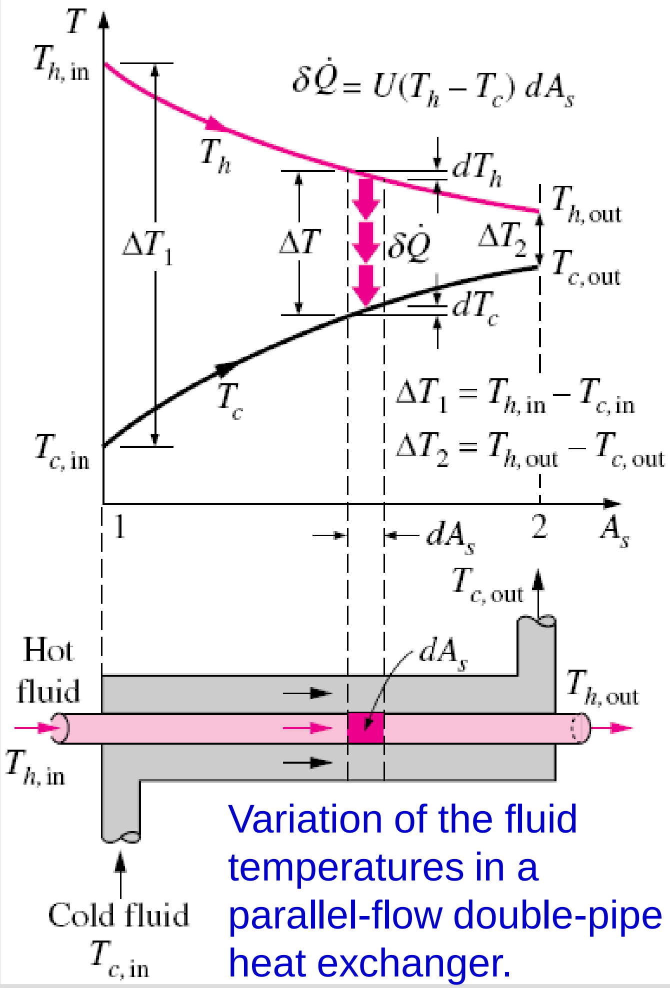
$$\frac{d(T_h - T_c)}{T_h - T_c} = -U\, dA_s\left(\frac{1}{\dot{m}_h c_{ph}} + \frac{1}{\dot{m}_c c_{pc}}\right)$$

$$\ln\frac{T_{h,\text{out}} - T_{c,\text{out}}}{T_{h,\text{in}} - T_{c,\text{in}}} = -UA_s\left(\frac{1}{\dot{m}_h c_{ph}} + \frac{1}{\dot{m}_c c_{pc}}\right)$$

$$\dot{Q} = UA_s\, \Delta T_{\text{lm}}$$

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$$

log mean temperature difference



Variation of the fluid temperatures in a parallel-flow double-pipe heat exchanger.

The arithmetic mean temperature difference

$$\Delta T_{am} = \frac{1}{2}(\Delta T_1 + \Delta T_2)$$

The logarithmic mean temperature difference ΔT_{lm} is an *exact* representation of the *average temperature difference* between the hot and cold fluids.

Note that ΔT_{lm} is always less than ΔT_{am}. Therefore, using ΔT_{am} in calculations instead of ΔT_{lm} will overestimate the rate of heat transfer in a heat exchanger between the two fluids.

When ΔT_1 differs from ΔT_2 by no more than 40 percent, the error in using the arithmetic mean temperature difference is less than 1 percent. But the error increases to undesirable levels when ΔT_1 differs from ΔT_2 by greater amounts.

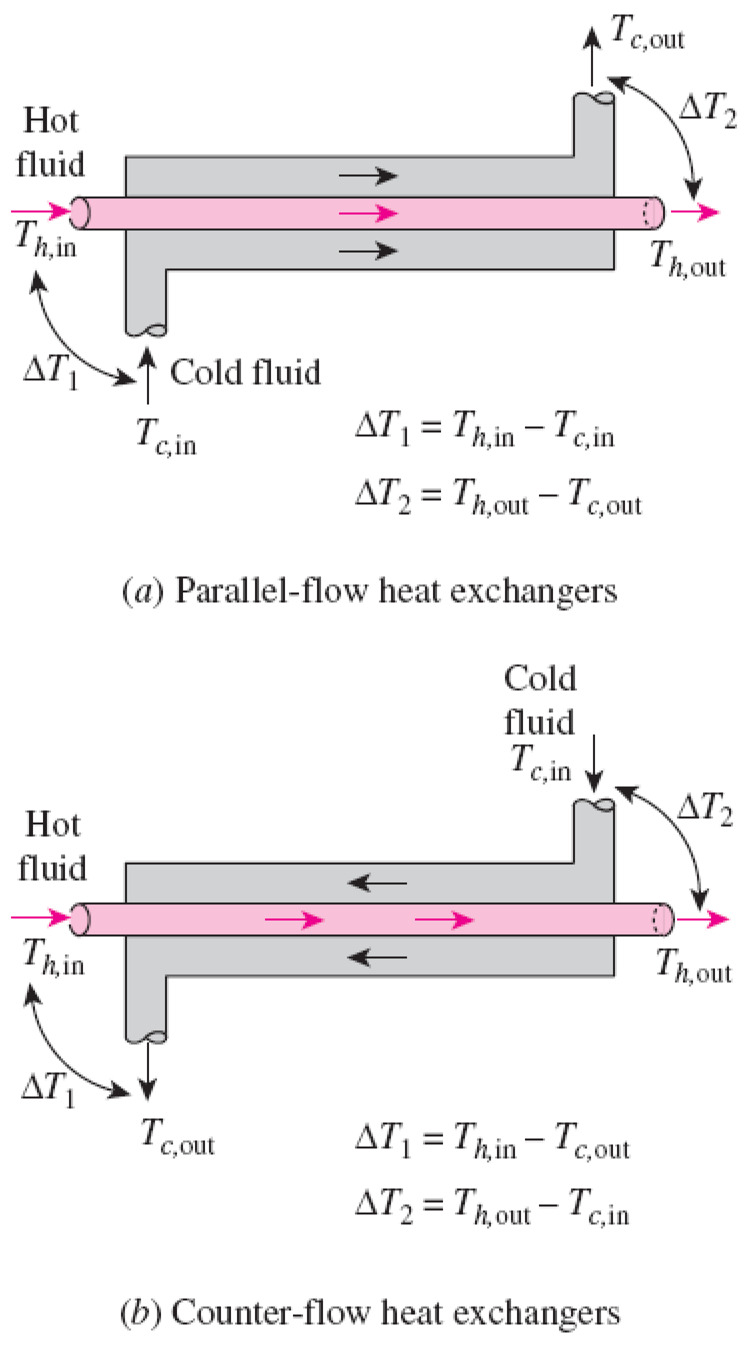


FIGURE 11–15
The ΔT_1 and ΔT_2 expressions in parallel-flow and counter-flow heat exchangers.

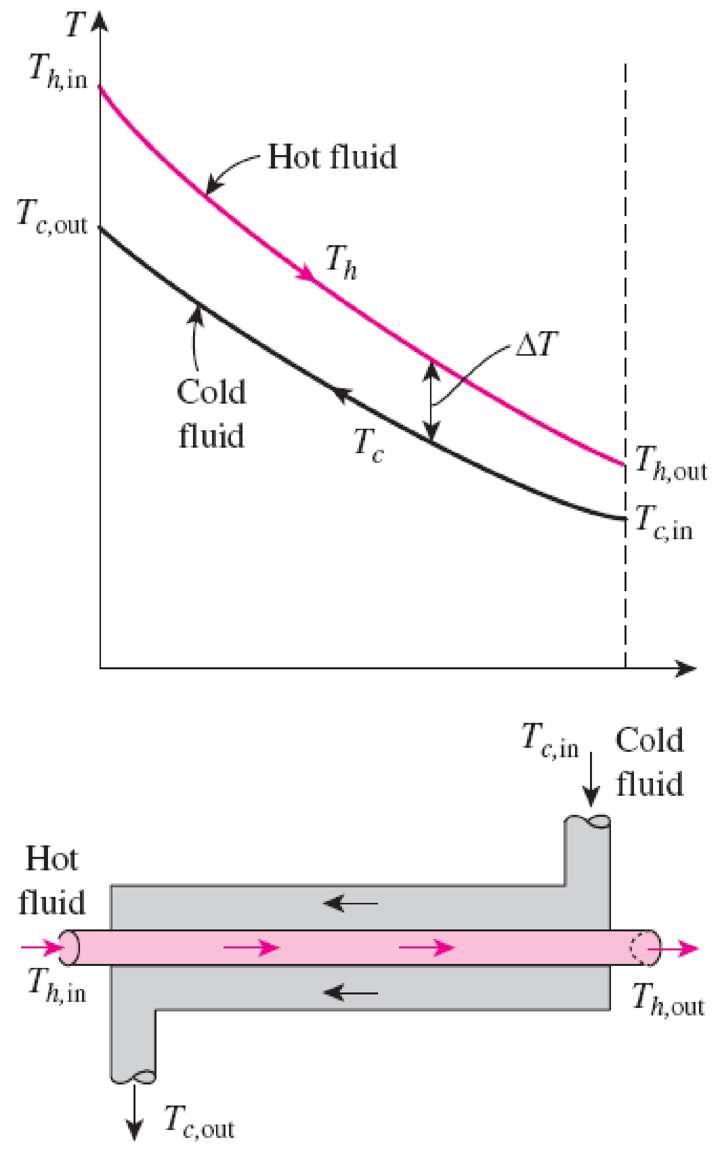


FIGURE 11–16
The variation of the fluid temperatures in a counter-flow double-pipe heat exchanger.

Counter-Flow Heat Exchangers

In the limiting case, the cold fluid will be heated to the inlet temperature of the hot fluid.

However, the outlet temperature of the cold fluid can *never* exceed the inlet temperature of the hot fluid.

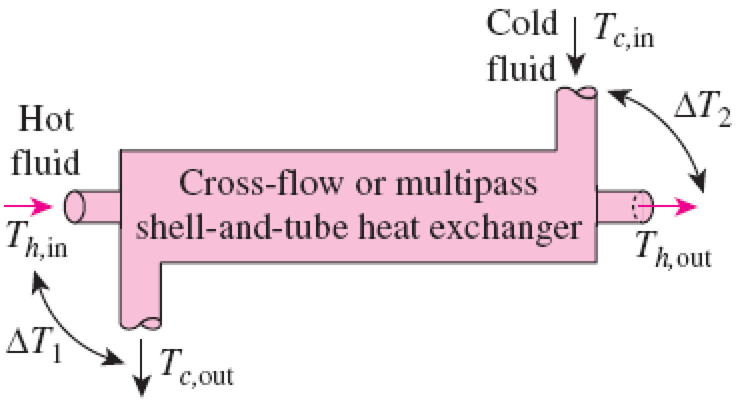
For specified inlet and outlet temperatures, ΔT_{lm} a counter-flow heat exchanger is always greater than that for a parallel-flow heat exchanger.

That is, $\boldsymbol{\Delta T_{lm,\,CF} > \Delta T_{lm,\,PF}}$, and thus a smaller surface area (and thus a smaller heat exchanger) is needed to achieve a specified heat transfer rate in a counter-flow heat exchanger.

When the *heat capacity rates* of the two fluids are *equal*

$$\Delta T_{lm} = \Delta T_1 = \Delta T_2$$

Multipass and Cross-Flow Heat Exchangers: Use of a Correction Factor



Heat transfer rate:

$$\dot{Q} = UA_s F \Delta T_{\text{lm,CF}}$$

where $\Delta T_{\text{lm,CF}} = \dfrac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$

$$\Delta T_1 = T_{h,\text{in}} - T_{c,\text{out}}$$

$$\Delta T_2 = T_{h,\text{out}} - T_{c,\text{in}}$$

and $F = \ldots$ (Fig. 11–18)

FIGURE 11–17
The determination of the heat transfer rate for cross-flow and multipass shell-and-tube heat exchangers using the correction factor.

$$\Delta T_{\text{lm}} = F\, \Delta T_{\text{lm, CF}}$$

***F* correction factor** depends on the *geometry* of the heat exchanger and the inlet and outlet temperatures of the hot and cold fluid streams.

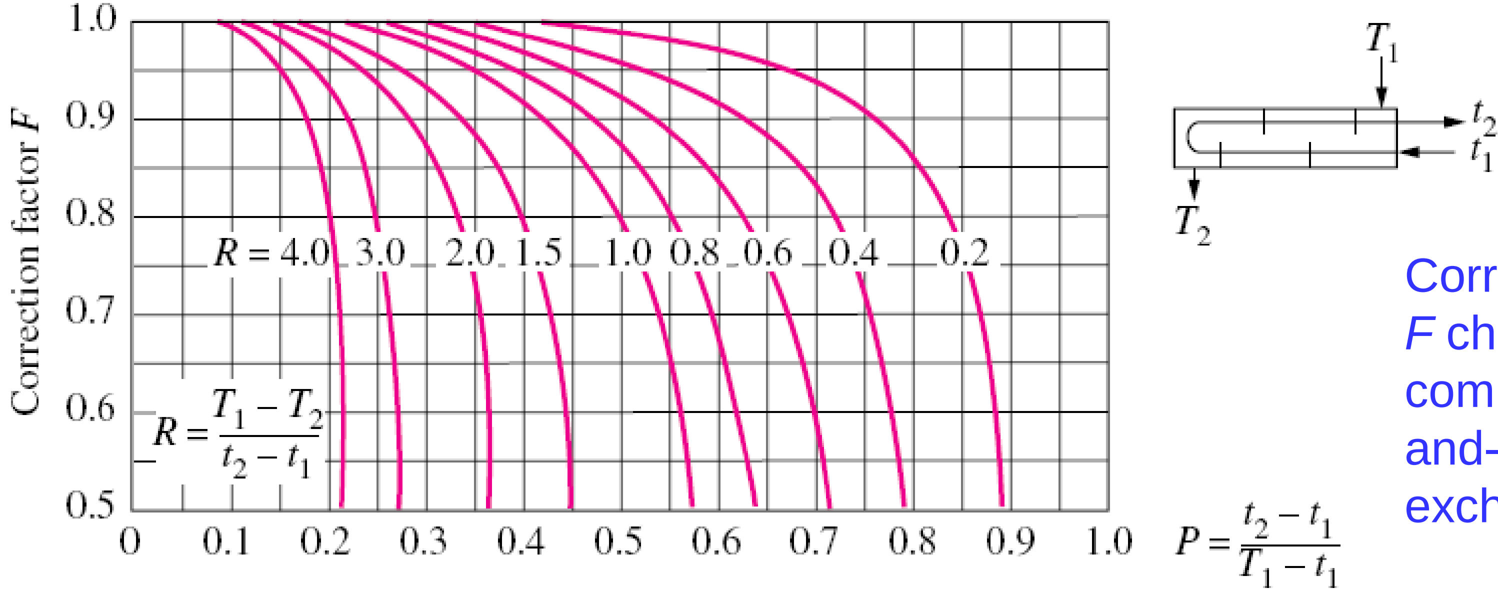
F for common cross-flow and shell-and-tube heat exchanger configurations is given in the figure versus two temperature ratios *P* and *R* defined as

$$P = \frac{t_2 - t_1}{T_1 - t_1}$$

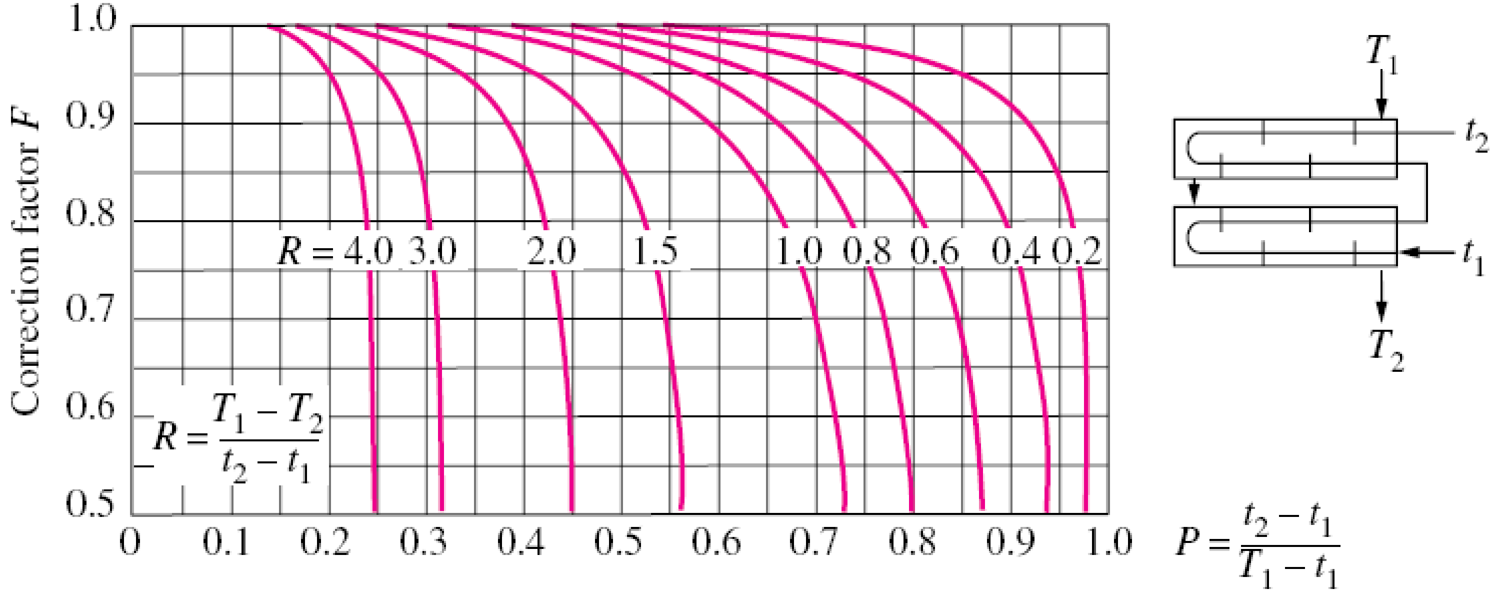
$$R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{(\dot{m}c_p)_{\text{tube side}}}{(\dot{m}c_p)_{\text{shell side}}}$$

1 and 2 *inlet* and *outlet*
T* and *t *shell*- and *tube-side* temperatures

***F* = 1 for a condenser or boiler**

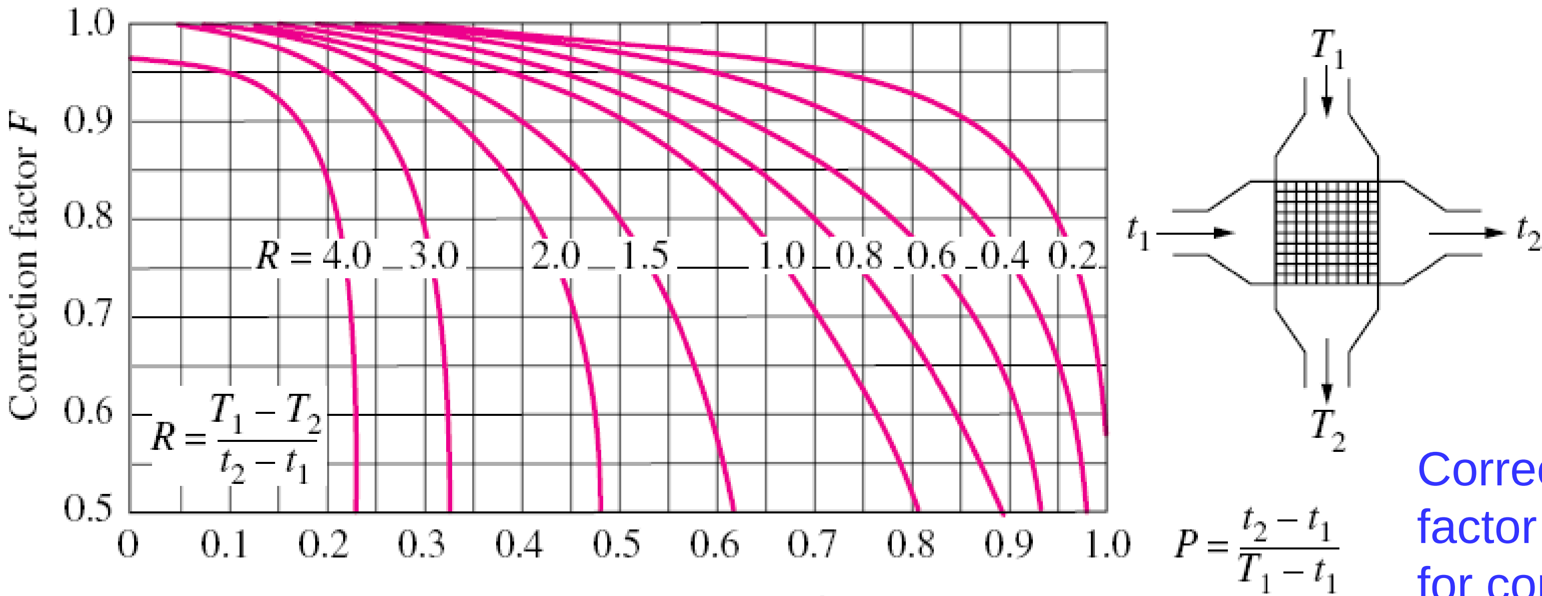


(*a*) One-shell pass and 2, 4, 6, etc. (any multiple of 2), tube passes

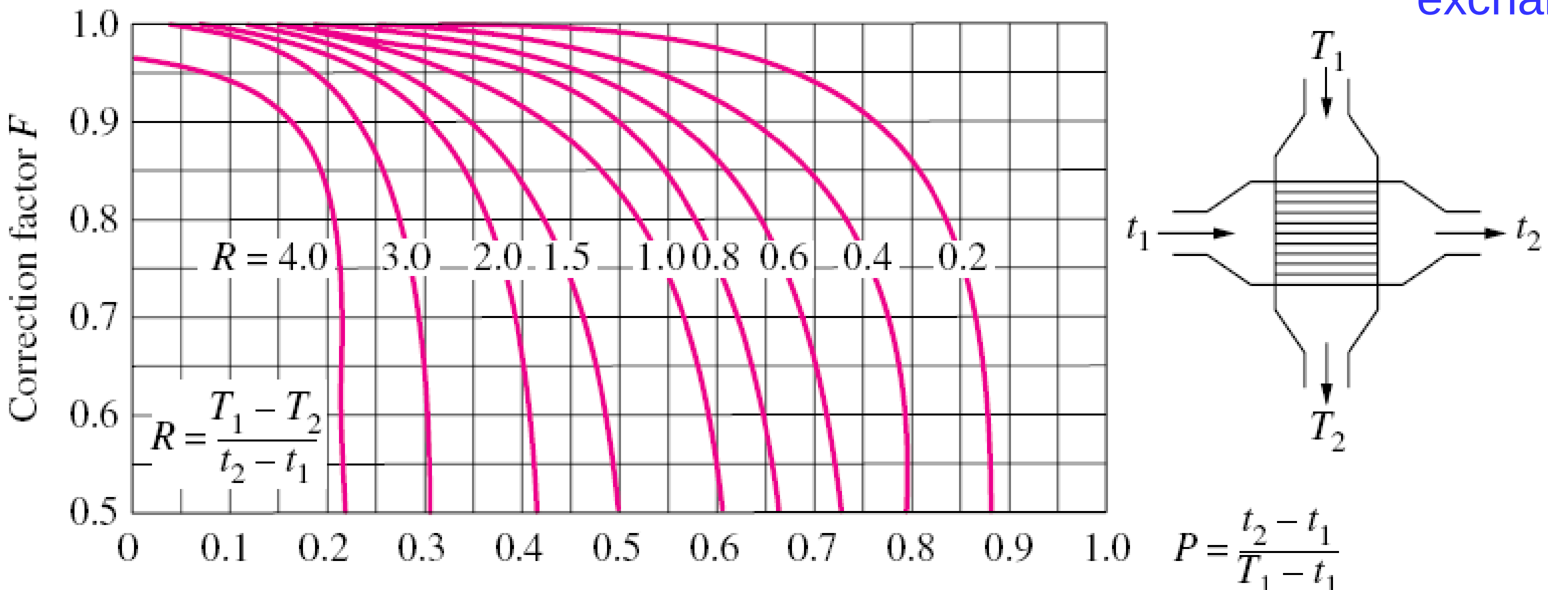


(*b*) Two-shell passes and 4, 8, 12, etc. (any multiple of 4), tube passes

Correction factor *F* charts for common shell-and-tube heat exchangers.



(*c*) Single-pass cross-flow with both fluids *unmixed*



(*d*) Single-pass cross-flow with one fluid *mixed* and the other *unmixed*

Correction factor *F* charts for common cross-flow heat exchangers.

The LMTD method is very suitable for determining the *size* of a heat exchanger to realize prescribed outlet temperatures when the mass flow rates and the inlet and outlet temperatures of the hot and cold fluids are specified.

With the LMTD method, the task is to ***select*** a heat exchanger that will meet the prescribed heat transfer requirements. The procedure to be followed by the selection process is:

1. Select the type of heat exchanger suitable for the application.
2. Determine any unknown inlet or outlet temperature and the heat transfer rate using an energy balance.
3. Calculate the log mean temperature difference ΔT_{lm} and the correction factor F, if necessary.
4. Obtain (select or calculate) the value of the overall heat transfer coefficient U.
5. Calculate the heat transfer surface area A_s .

The task is completed by selecting a heat exchanger that has a heat transfer surface area equal to or larger than A_s.

EXAMPLE 13–3 The Condensation of Steam in a Condenser

Steam in the condenser of a power plant is to be condensed at a temperature of 30°C with cooling water from a nearby lake, which enters the tubes of the condenser at 14°C and leaves at 22°C. The surface area of the tubes is 45 m^2, and the overall heat transfer coefficient is 2100 $W/m^2 \cdot °C$. Determine the mass flow rate of the cooling water needed and the rate of condensation of the steam in the condenser.

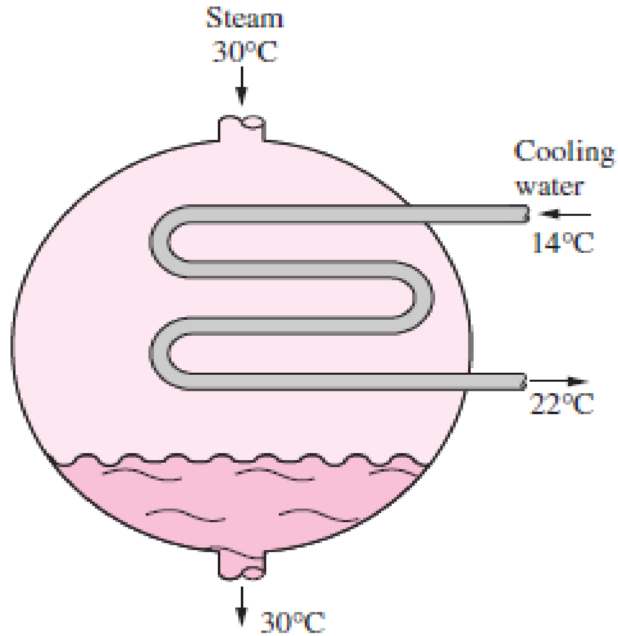


FIGURE 13–19
Schematic for Example 13–3.

SOLUTION Steam is condensed by cooling water in the condenser of a power plant. The mass flow rate of the cooling water and the rate of condensation are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant.

Properties The heat of vaporization of water at 30°C is h_{fg} = 2431 kJ/kg and the specific heat of cold water at the average temperature of 18°C is C_p = 4184 J/kg · °C (Table A–9).

Analysis The schematic of the condenser is given in Figure 13–19. The condenser can be treated as a counter-flow heat exchanger since the temperature of one of the fluids (the steam) remains constant.

The temperature difference between the steam and the cooling water at the two ends of the condenser is

$$\Delta T_1 = T_{h,\,\text{in}} - T_{c,\,\text{out}} = (30 - 22)°\text{C} = 8°\text{C}$$
$$\Delta T_2 = T_{h,\,\text{out}} - T_{c,\,\text{in}} = (30 - 14)°\text{C} = 16°\text{C}$$

That is, the temperature difference between the two fluids varies from 8°C at one end to 16°C at the other. The proper average temperature difference between the two fluids is the *logarithmic mean temperature difference* (not the arithmetic), which is determined from

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{8 - 16}{\ln(8/16)} = 11.5°\text{C}$$

This is a little less than the arithmetic mean temperature difference of $\frac{1}{2}(8 + 16)$ = 12°C. Then the heat transfer rate in the condenser is determined from

$$\dot{Q} = UA_s \Delta T_{lm} = (2100 \text{ W/m}^2 \cdot °\text{C})(45 \text{ m}^2)(11.5°\text{C}) = \mathbf{1.087 \times 10^6 \text{ W} = 1087 \text{ kW}}$$

Therefore, the steam will lose heat at a rate of 1,087 kW as it flows through the condenser, and the cooling water will gain practically all of it, since the condenser is well insulated.

The mass flow rate of the cooling water and the rate of the condensation of the steam are determined from $\dot{Q} = [\dot{m}C_p(T_{out} - T_{in})]_{cooling\ water} = (\dot{m}h_{fg})_{steam}$ to be

$$\dot{m}_{cooling\ water} = \frac{\dot{Q}}{C_p(T_{out} - T_{in})} = \frac{1{,}087 \text{ kJ/s}}{(4.184 \text{ kJ/kg} \cdot °\text{C})(22 - 14)°\text{C}} = \mathbf{32.5 \text{ kg/s}}$$

and

$$\dot{m}_{steam} = \frac{\dot{Q}}{h_{fg}} = \frac{1{,}087 \text{ kJ/s}}{2431 \text{ kJ/kg}} = \mathbf{0.45 \text{ kg/s}}$$

Therefore, we need to circulate about 72 kg of cooling water for each 1 kg of steam condensing to remove the heat released during the condensation process.

EXAMPLE 13–4 Heating Water in a Counter-Flow Heat Exchanger

A counter-flow double-pipe heat exchanger is to heat water from 20°C to 80°C at a rate of 1.2 kg/s. The heating is to be accomplished by geothermal water available at 160°C at a mass flow rate of 2 kg/s. The inner tube is thin-walled and has a diameter of 1.5 cm. If the overall heat transfer coefficient of the heat exchanger is 640 W/m^2 · °C, determine the length of the heat exchanger required to achieve the desired heating.

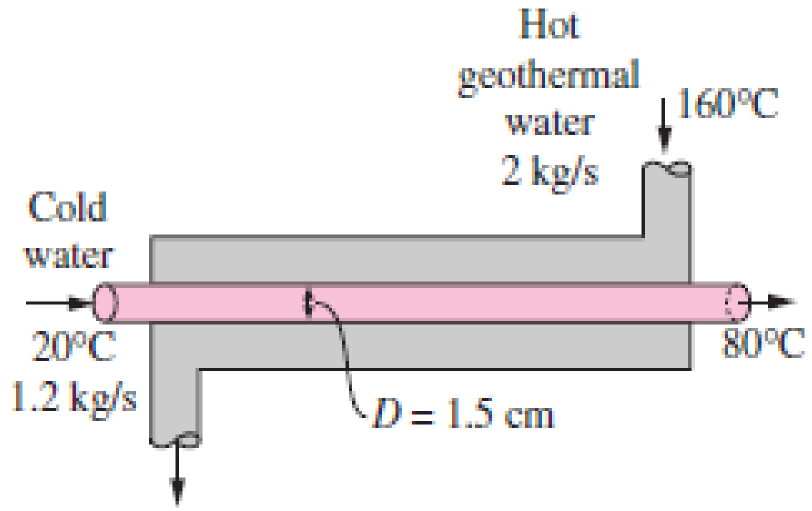


FIGURE 13–20
Schematic for Example 13–4.

SOLUTION Water is heated in a counter-flow double-pipe heat exchanger by geothermal water. The required length of the heat exchanger is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant.

Properties We take the specific heats of water and geothermal fluid to be 4.18 and 4.31 kJ/kg · °C, respectively.

Analysis The schematic of the heat exchanger is given in Figure 13–20. The rate of heat transfer in the heat exchanger can be determined from

$$\dot{Q} = [\dot{m}C_p(T_{out} - T_{in})]_{water} = (1.2 \text{ kg/s})(4.18 \text{ kJ/kg} \cdot {}^\circ\text{C})(80 - 20)^\circ\text{C} = 301 \text{ kW}$$

Noting that all of this heat is supplied by the geothermal water, the outlet temperature of the geothermal water is determined to be

$$\dot{Q} = [\dot{m}C_p(T_{in} - T_{out})]_{geothermal} \longrightarrow T_{out} = T_{in} - \frac{\dot{Q}}{\dot{m}C_p}$$

$$= 160^\circ\text{C} - \frac{301 \text{ kW}}{(2 \text{ kg/s})(4.31 \text{ kJ/kg} \cdot {}^\circ\text{C})}$$

$$= 125^\circ\text{C}$$

Knowing the inlet and outlet temperatures of both fluids, the logarithmic mean temperature difference for this counter-flow heat exchanger becomes

$$\Delta T_1 = T_{h,\,in} - T_{c,\,out} = (160 - 80)^\circ\text{C} = 80^\circ\text{C}$$
$$\Delta T_2 = T_{h,\,out} - T_{c,\,in} = (125 - 20)^\circ\text{C} = 105^\circ\text{C}$$

and

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{80 - 105}{\ln(80/105)} = 92.0^\circ\text{C}$$

Then the surface area of the heat exchanger is determined to be

$$\dot{Q} = UA_s\,\Delta T_{lm} \longrightarrow A_s = \frac{\dot{Q}}{U\,\Delta T_{lm}} = \frac{301{,}000 \text{ W}}{(640 \text{ W/m}^2 \cdot {}^\circ\text{C})(92.0^\circ\text{C})} = 5.11 \text{ m}^2$$

To provide this much heat transfer surface area, the length of the tube must be

$$A_s = \pi DL \longrightarrow L = \frac{A_s}{\pi D} = \frac{5.11 \text{ m}^2}{\pi(0.015 \text{ m})} = 108 \text{ m}$$

EXAMPLE 13–5 **Heating of Glycerin in a Multipass Heat Exchanger**

A 2-shell passes and 4-tube passes heat exchanger is used to heat glycerin from 20°C to 50°C by hot water, which enters the thin-walled 2-cm-diameter tubes at 80°C and leaves at 40°C (Fig. 13–21). The total length of the tubes in the heat exchanger is 60 m. The convection heat transfer coefficient is 25 $W/m^2 \cdot °C$ on the glycerin (shell) side and 160 $W/m^2 \cdot °C$ on the water (tube) side. Determine the rate of heat transfer in the heat exchanger (*a*) before any fouling occurs and (*b*) after fouling with a fouling factor of 0.0006 $m^2 \cdot °C/W$ occurs on the outer surfaces of the tubes.

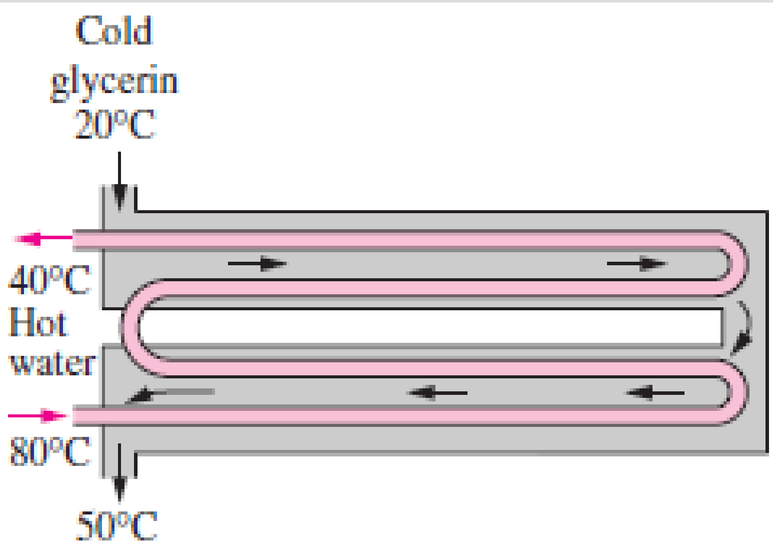


FIGURE 13–21
Schematic for Example 13–5.

SOLUTION Glycerin is heated in a 2-shell passes and 4-tube passes heat exchanger by hot water. The rate of heat transfer for the cases of fouling and no fouling are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Heat transfer coefficients and fouling factors are constant and uniform. **5** The thermal resistance of the inner tube is negligible since the tube is thin-walled and highly conductive.

Analysis The tubes are said to be thin-walled, and thus it is reasonable to assume the inner and outer surface areas of the tubes to be equal. Then the heat transfer surface area becomes

$$A_s = \pi DL = \pi(0.02 \text{ m})(60 \text{ m}) = 3.77 \text{ m}^2$$

The rate of heat transfer in this heat exchanger can be determined from

$$\dot{Q} = UA_s F \, \Delta T_{\text{lm}, CF}$$

where F is the correction factor and $\Delta T_{\text{lm}, CF}$ is the log mean temperature difference for the counter-flow arrangement. These two quantities are determined from

$$\Delta T_1 = T_{h,\text{in}} - T_{c,\text{out}} = (80 - 50)^\circ\text{C} = 30^\circ\text{C}$$

$$\Delta T_2 = T_{h,\text{out}} - T_{c,\text{in}} = (40 - 20)^\circ\text{C} = 20^\circ\text{C}$$

$$\Delta T_{\text{lm}, CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{30 - 20}{\ln(30/20)} = 24.7^\circ\text{C}$$

and

$$\left.\begin{aligned} P &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{40 - 80}{20 - 80} = 0.67 \\ R &= \frac{T_1 - T_2}{t_2 - t_1} = \frac{20 - 50}{40 - 80} = 0.75 \end{aligned}\right\} F = 0.91 \qquad \text{(Fig. 13–18}b\text{)}$$

(a) In the case of no fouling, the overall heat transfer coefficient U is determined from

$$U = \frac{1}{\dfrac{1}{h_i} + \dfrac{1}{h_o}} = \frac{1}{\dfrac{1}{160\ \text{W/m}^2\cdot{}^\circ\text{C}} + \dfrac{1}{25\ \text{W/m}^2\cdot{}^\circ\text{C}}} = 21.6\ \text{W/m}^2\cdot{}^\circ\text{C}$$

Then the rate of heat transfer becomes

$$\dot{Q} = UA_s F\,\Delta T_{\text{lm, }CF} = (21.6\ \text{W/m}^2\cdot{}^\circ\text{C})(3.77\text{m}^2)(0.91)(24.7^\circ\text{C}) = \mathbf{1830\ W}$$

(b) When there is fouling on one of the surfaces, the overall heat transfer coefficient U is

$$U = \frac{1}{\dfrac{1}{h_i} + \dfrac{1}{h_o} + R_f} = \frac{1}{\dfrac{1}{160\ \text{W/m}^2\cdot{}^\circ\text{C}} + \dfrac{1}{25\ \text{W/m}^2\cdot{}^\circ\text{C}} + 0.0006\ \text{m}^2\cdot{}^\circ\text{C/W}}$$

$$= 21.3\ \text{W/m}^2\cdot{}^\circ\text{C}$$

The rate of heat transfer in this case becomes

$$\dot{Q} = UA_s F\,\Delta T_{\text{lm, }CF} = (21.3\ \text{W/m}^2\cdot{}^\circ\text{C})(3.77\ \text{m}^2)(0.91)(24.7^\circ\text{C}) = \mathbf{1805\ W}$$

EXAMPLE 13–6 Cooling of an Automotive Radiator

A test is conducted to determine the overall heat transfer coefficient in an automotive radiator that is a compact cross-flow water-to-air heat exchanger with both fluids (air and water) unmixed (Fig. 13–22). The radiator has 40 tubes of internal diameter 0.5 cm and length 65 cm in a closely spaced plate-finned matrix. Hot water enters the tubes at 90°C at a rate of 0.6 kg/s and leaves at 65°C. Air flows across the radiator through the interfin spaces and is heated from 20°C to 40°C. Determine the overall heat transfer coefficient U_i of this radiator based on the inner surface area of the tubes.

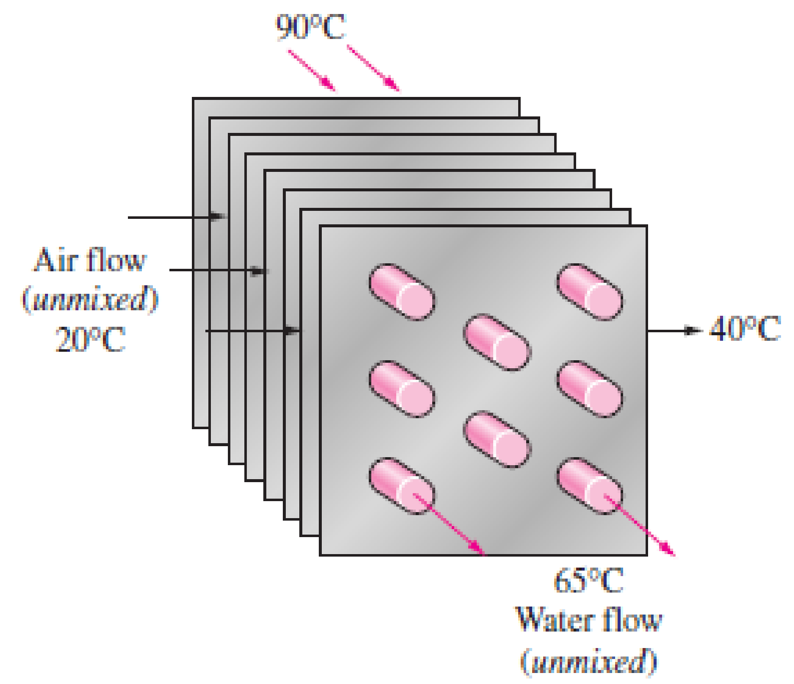


FIGURE 13–22
Schematic for Example 13–6.

SOLUTION During an experiment involving an automotive radiator, the inlet and exit temperatures of water and air and the mass flow rate of water are measured. The overall heat transfer coefficient based on the inner surface area is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Changes in the kinetic and potential energies of fluid streams are negligible. **3** Fluid properties are constant.

Properties The specific heat of water at the average temperature of (90 + 65)/2 = 77.5°C is 4.195 kJ/kg · °C.

Analysis The rate of heat transfer in this radiator from the hot water to the air is determined from an energy balance on water flow,

$$\dot{Q} = [\dot{m}C_p(T_{in} - T_{out})]_{water} = (0.6 \text{ kg/s})(4.195 \text{ kJ/kg} \cdot {}^\circ\text{C})(90 - 65)^\circ\text{C} = 62.93 \text{ kW}$$

The tube-side heat transfer area is the total surface area of the tubes, and is determined from

$$A_i = n\pi D_i L = (40)\pi(0.005 \text{ m})(0.65 \text{ m}) = 0.408 \text{ m}^2$$

Knowing the rate of heat transfer and the surface area, the overall heat transfer coefficient can be determined from

$$\dot{Q} = U_i A_i F \Delta T_{lm, CF} \longrightarrow U_i = \frac{\dot{Q}}{A_i F \Delta T_{lm, CF}}$$

where F is the correction factor and $\Delta T_{lm, CF}$ is the log mean temperature difference for the counter-flow arrangement. These two quantities are found to be

$$\Delta T_1 = T_{h, in} - T_{c, out} = (90 - 40)^\circ\text{C} = 50^\circ\text{C}$$

$$\Delta T_2 = T_{h, out} - T_{c, in} = (65 - 20)^\circ\text{C} = 45^\circ\text{C}$$

$$\Delta T_{lm, CF} = \frac{\Delta T_1 - \Delta T_2}{\ln (\Delta T_1/\Delta T_2)} = \frac{50 - 45}{\ln (50/45)} = 47.6^\circ\text{C}$$

and

$$\left.\begin{aligned} P &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{65 - 90}{20 - 90} = 0.36 \\ R &= \frac{T_1 - T_2}{t_2 - t_1} = \frac{20 - 40}{65 - 90} = 0.80 \end{aligned}\right\} F = 0.97 \qquad \text{(Fig. 13–18}c\text{)}$$

Substituting, the overall heat transfer coefficient U_i is determined to be

$$U_i = \frac{\dot{Q}}{A_i F \Delta T_{\text{lm, }CF}} = \frac{62{,}930 \text{ W}}{(0.408 \text{ m}^2)(0.97)(47.6°\text{C})} = \mathbf{3341 \text{ W/m}^2 \cdot °\text{C}}$$

Note that the overall heat transfer coefficient on the air side will be much lower because of the large surface area involved on that side.

THE EFFECTIVENESS–NTU METHOD

A second kind of problem encountered in heat exchanger analysis is the determination of the *heat transfer rate* and the *outlet temperatures* of the hot and cold fluids for prescribed fluid mass flow rates and inlet temperatures when the *type* and *size* of the heat exchanger are specified.

Heat transfer effectiveness

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{max}} = \frac{\text{Actual heat transfer rate}}{\text{Maximum possible heat transfer rate}}$$

$$\dot{Q} = C_c(T_{c,\,out} - T_{c,\,in}) = C_h(T_{h,\,in} - T_{h,\,out})$$

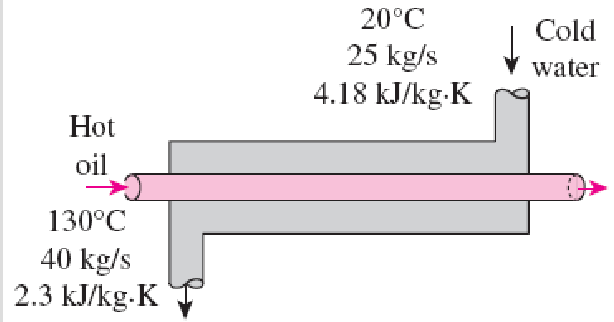
$$C_c = \dot{m}_c c_{pc} \text{ and } C_h = \dot{m}_c c_{ph}$$

$$\Delta T_{max} = T_{h,\,in} - T_{c,\,in}$$

$$\dot{Q}_{max} = C_{min}(T_{h,\,in} - T_{c,\,in})$$

the maximum possible heat transfer rate

C_{min} is the smaller of C_h and C_c



$$C_c = \dot{m}_c c_{pc} = 104.5 \text{ kW/K}$$

$$C_h = \dot{m}_c c_{ph} = 92 \text{ kW/K}$$

$$C_{min} = 92 \text{ kW/K}$$

$$\Delta T_{max} = T_{h,in} - T_{c,in} = 110°C$$

$$\dot{Q}_{max} = C_{min}\,\Delta T_{max} = 10{,}120 \text{ kW}$$

FIGURE 11–23
The determination of the maximum rate of heat transfer in a heat exchanger.

EXAMPLE 13–7 Upper Limit for Heat Transfer in a Heat Exchanger

Cold water enters a counter-flow heat exchanger at 10°C at a rate of 8 kg/s, where it is heated by a hot water stream that enters the heat exchanger at 70°C at a rate of 2 kg/s. Assuming the specific heat of water to remain constant at $C_p = 4.18$ kJ/kg · °C, determine the maximum heat transfer rate and the outlet temperatures of the cold and the hot water streams for this limiting case.

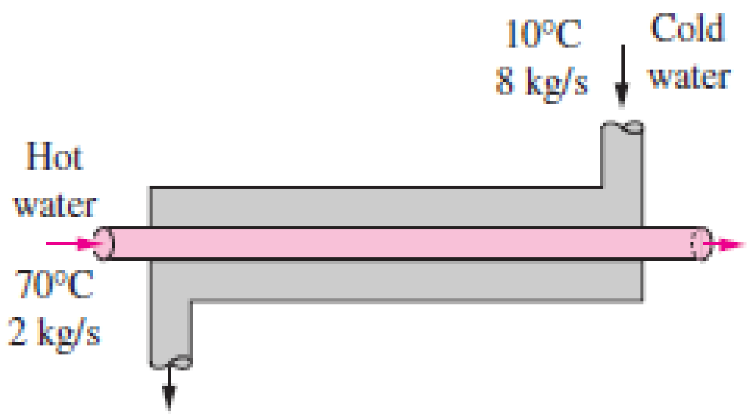


FIGURE 13–24
Schematic for Example 13–7.

SOLUTION Cold and hot water streams enter a heat exchanger at specified temperatures and flow rates. The maximum rate of heat transfer in the heat exchanger is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Heat transfer coefficients and fouling factors are constant and uniform. **5** The thermal resistance of the inner tube is negligible since the tube is thin-walled and highly conductive.

Properties The specific heat of water is given to be $C_p = 4.18$ kJ/kg · °C.

Analysis A schematic of the heat exchanger is given in Figure 13–24. The heat capacity rates of the hot and cold fluids are determined from

$$C_h = \dot{m}_h C_{ph} = (2 \text{ kg/s})(4.18 \text{ kJ/kg} \cdot {}^\circ\text{C}) = 8.36 \text{ kW/}^\circ\text{C}$$

and

$$C_c = \dot{m}_c C_{pc} = (8 \text{ kg/s})(4.18 \text{ kJ/kg} \cdot {}^\circ\text{C}) = 33.4 \text{ kW/}^\circ\text{C}$$

Therefore

$$C_{\min} = C_h = 8.36 \text{ kW/}^\circ\text{C}$$

which is the smaller of the two heat capacity rates. Then the maximum heat transfer rate is determined from Eq. 13-32 to be

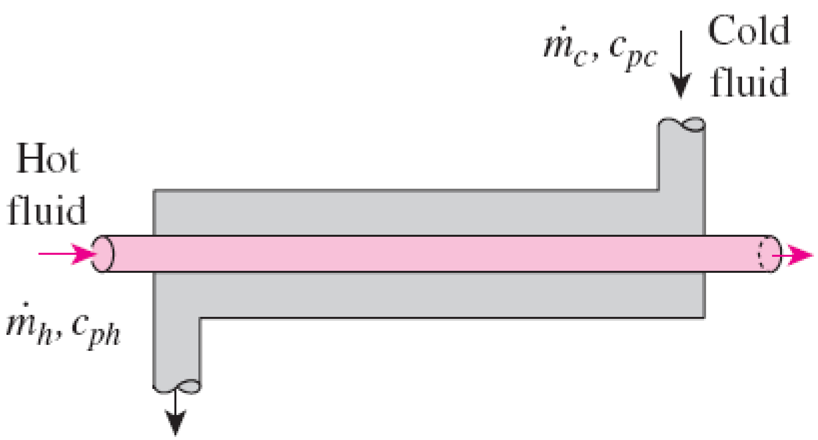
$$\begin{aligned}\dot{Q}_{\max} &= C_{\min}(T_{h,\text{ in}} - T_{c,\text{ in}}) \\ &= (8.36 \text{ kW/}^\circ\text{C})(70 - 10)^\circ\text{C} \\ &= 502 \text{ kW}\end{aligned}$$

That is, the maximum possible heat transfer rate in this heat exchanger is 502 kW. This value would be approached in a counter-flow heat exchanger with a *very large* heat transfer surface area.

The maximum temperature difference in this heat exchanger is $\Delta T_{max} = T_{h,\,in} - T_{c,\,in} = (70 - 10)°C = 60°C$. Therefore, the hot water cannot be cooled by more than 60°C (to 10°C) in this heat exchanger, and the cold water cannot be heated by more than 60°C (to 70°C), no matter what we do. The outlet temperatures of the cold and the hot streams in this limiting case are determined to be

$$\dot{Q} = C_c(T_{c,\,out} - T_{c,\,in}) \longrightarrow T_{c,\,out} = T_{c,\,in} + \frac{\dot{Q}}{C_c} = 10°C + \frac{502\text{ kW}}{33.4\text{ kW/°C}} = 25°C$$

$$\dot{Q} = C_h(T_{h,\,in} - T_{h,\,out}) \longrightarrow T_{h,\,out} = T_{h,\,in} - \frac{\dot{Q}}{C_h} = 70°C - \frac{502\text{ kW}}{8.38\text{ kW/°C}} = 10°C$$



$$\dot{Q} = \dot{m}_h \, c_{ph} \, \Delta T_h$$
$$= \dot{m}_c \, c_{pc} \, \Delta T_c$$

If $\quad \dot{m}_c \, c_{pc} = \dot{m}_h \, c_{ph}$

then $\quad \Delta T_h = \Delta T_c$

FIGURE 11–25
The temperature rise of the cold fluid in a heat exchanger will be equal to the temperature drop of the hot fluid when the heat capacity rates of the hot and cold fluids are identical.

Actual heat transfer rate

$$\dot{Q} = \varepsilon \dot{Q}_{\max} = \varepsilon C_{\min}(T_{h,\,\mathrm{in}} - T_{c,\,\mathrm{in}})$$

if $C_c = C_{\min}$:

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{C_c(T_{c,\mathrm{out}} - T_{c,\mathrm{in}})}{C_c(T_{h,\mathrm{in}} - T_{c,\mathrm{in}})} = \frac{T_{c,\mathrm{out}} - T_{c,\mathrm{in}}}{T_{h,\mathrm{in}} - T_{c,\mathrm{in}}}$$

if $C_h = C_{\min}$:

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{C_h(T_{h,\mathrm{in}} - T_{h,\mathrm{out}})}{C_h(T_{h,\mathrm{in}} - T_{c,\mathrm{in}})} = \frac{T_{h,\mathrm{out}} - T_{h,\mathrm{out}}}{T_{h,\mathrm{in}} - T_{c,\mathrm{in}}}$$

The effectiveness of a heat exchanger depends on the *geometry* of the heat exchanger as well as the *flow arrangement*.

Therefore, different types of heat exchangers have different effectiveness relations.

We illustrate the development of the effectiveness e relation for the double-pipe *parallel-flow* heat exchanger.

$$\ln \frac{T_{h,\text{out}} - T_{c,\text{out}}}{T_{h,\text{in}} - T_{c,\text{in}}} = -\frac{UA_s}{C_c}\left(1 + \frac{C_c}{C_h}\right)$$

$$T_{h,\text{out}} = T_{h,\text{in}} - \frac{C_c}{C_h}(T_{c,\text{out}} - T_{c,\text{in}})$$

$$\ln \frac{T_{h,\text{in}} - T_{c,\text{in}} + T_{c,\text{in}} - T_{c,\text{out}} - \frac{C_c}{C_h}(T_{c,\text{out}} - T_{c,\text{in}})}{T_{h,\text{in}} - T_{c,\text{in}}} = -\frac{UA_s}{C_c}\left(1 + \frac{C_c}{C_h}\right)$$

$$\ln \left[1 - \left(1 + \frac{C_c}{C_h}\right)\frac{T_{c,\text{out}} - T_{c,\text{in}}}{T_{h,\text{in}} - T_{c,\text{in}}}\right] = -\frac{UA_s}{C_c}\left(1 + \frac{C_c}{C_h}\right)$$

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{C_c(T_{c,\text{out}} - T_{c,\text{in}})}{C_{\min}(T_{h,\text{in}} - T_{c,\text{in}})} \longrightarrow \frac{T_{c,\text{out}} - T_{c,\text{in}}}{T_{h,\text{in}} - T_{c,\text{in}}} = \varepsilon \frac{C_{\min}}{C_c}$$

$$\varepsilon_{\text{parallel flow}} = \frac{1 - \exp\left[-\frac{UA_s}{C_c}\left(1 + \frac{C_c}{C_h}\right)\right]}{\left(1 + \frac{C_c}{C_h}\right)\frac{C_{\min}}{C_c}}$$

$$\varepsilon_{\text{parallel flow}} = \frac{1 - \exp\left[-\frac{UA_s}{C_{\min}}\left(1 + \frac{C_{\min}}{C_{\max}}\right)\right]}{1 + \frac{C_{\min}}{C_{\max}}}$$

Effectiveness relations of the heat exchangers typically involve the *dimensionless* group UA_s / C_{min}.

This quantity is called the **number of transfer units NTU**.

$$\text{NTU} = \frac{UA_s}{C_{\min}} = \frac{UA_s}{(\dot{m}c_p)_{\min}}$$

For specified values of U and C_{min}, the value of NTU *is a measure of the surface area A_s*. Thus, the larger the NTU, the larger the heat exchanger.

$$c = \frac{C_{\min}}{C_{\max}}$$

capacity ratio

The effectiveness of a heat exchanger is a function of the number of transfer units NTU and the capacity ratio *c*.

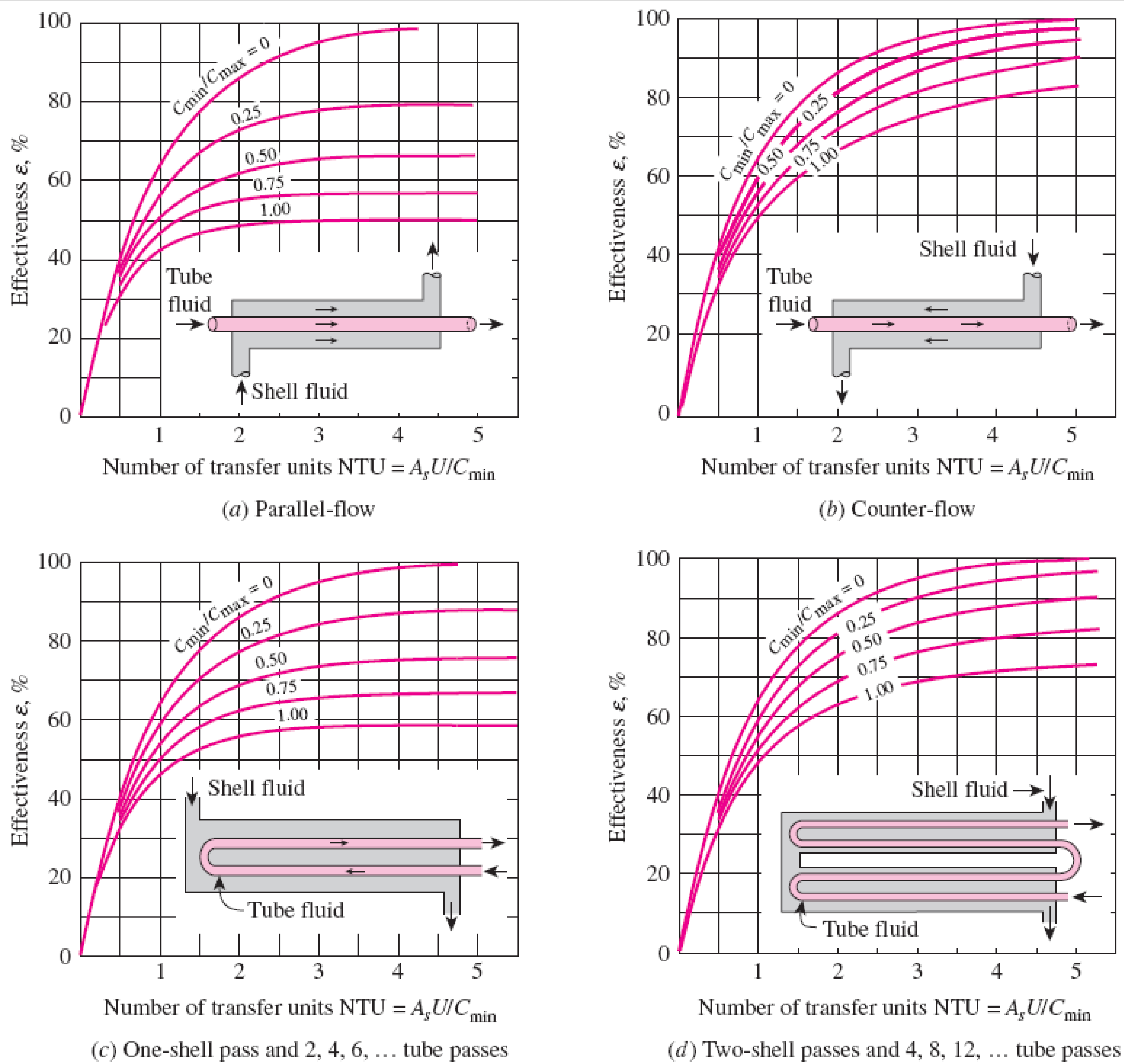
$$\varepsilon = \text{function}\ (UA_s/C_{\min}, C_{\min}/C_{\max}) = \text{function}\ (\text{NTU}, c)$$

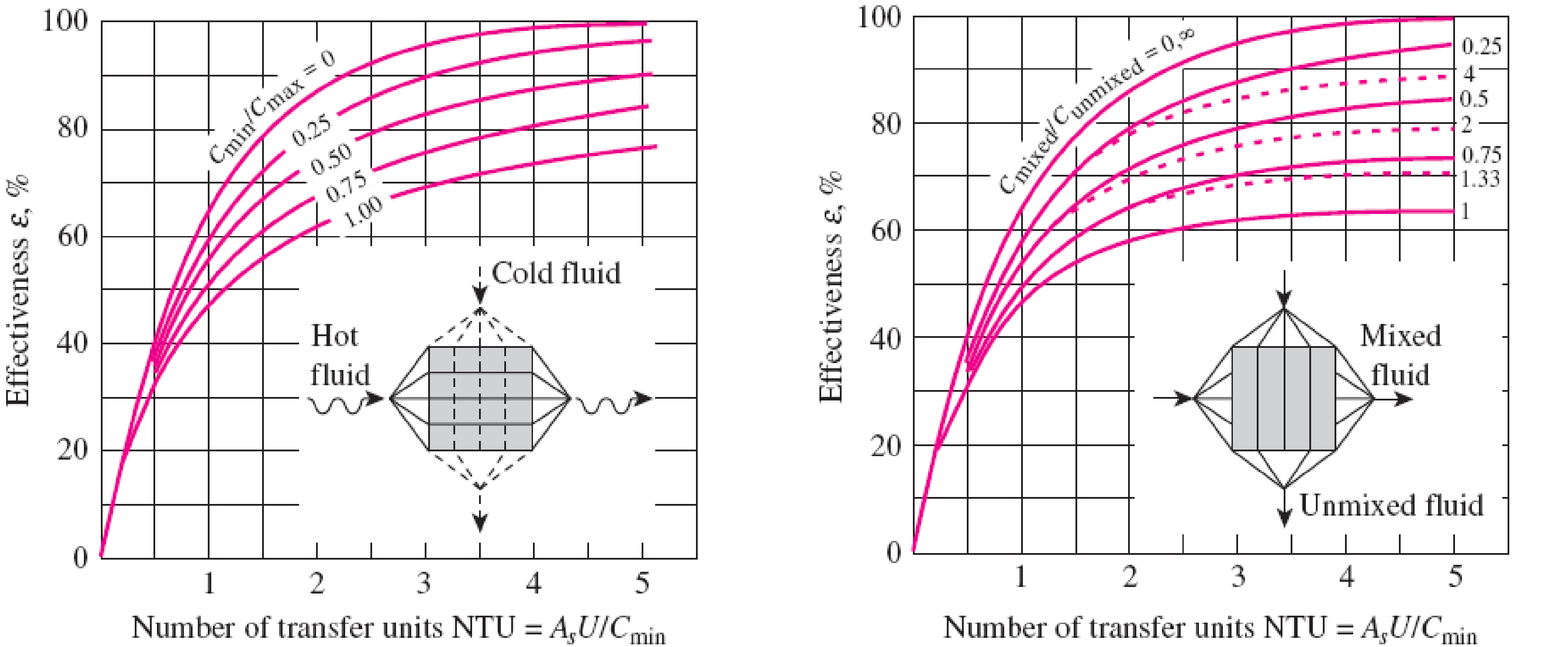
TABLE 11–4

Effectiveness relations for heat exchangers: NTU = UA_s/C_{min} and $c = C_{min}/C_{max} = (\dot{m}c_p)_{min}/(\dot{m}c_p)_{max}$

Heat exchanger type	Effectiveness relation
1 *Double pipe:*	
Parallel-flow	$\varepsilon = \dfrac{1 - \exp[-\text{NTU}(1 + c)]}{1 + c}$
Counter-flow	$\varepsilon = \dfrac{1 - \exp[-\text{NTU}(1 - c)]}{1 - c\exp[-\text{NTU}(1 - c)]}$
2 *Shell-and-tube:* One-shell pass 2, 4,…tube passes	$\varepsilon = 2\left\{1 + c + \sqrt{1 + c^2}\dfrac{1 + \exp[-\text{NTU}\sqrt{1 + c^2}]}{1 - \exp[-\text{NTU}\sqrt{1 + c^2}]}\right\}^{-1}$
3 *Cross-flow (single-pass)* Both fluids unmixed	$\varepsilon = 1 - \exp\left\{\dfrac{\text{NTU}^{0.22}}{c}[\exp(-c\,\text{NTU}^{0.78}) - 1]\right\}$
C_{max} mixed, C_{min} unmixed	$\varepsilon = \dfrac{1}{c}(1 - \exp\{-c[1 - \exp(-\text{NTU})]\})$
C_{min} mixed, C_{max} unmixed	$\varepsilon = 1 - \exp\left\{-\dfrac{1}{c}[1 - \exp(-c\,\text{NTU})]\right\}$
4 *All heat exchangers with $c = 0$*	$\varepsilon = 1 - \exp(-\text{NTU})$

Effectiveness for heat exchangers.





(*e*) Cross-flow with both fluids unmixed

(*f*) Cross-flow with one fluid mixed and the other unmixed

FIGURE 11–26

Effectiveness for heat exchangers.

TABLE 11–5

NTU relations for heat exchangers: $NTU = UA_s/C_{min}$ and $c = C_{min}/C_{max} = (\dot{m}c_p)_{min}/(\dot{m}c_p)_{max}$

Heat exchanger type	NTU relation
1 *Double-pipe:*	
Parallel-flow	$NTU = -\dfrac{\ln[1-\varepsilon(1+c)]}{1+c}$
Counter-flow	$NTU = \dfrac{1}{c-1}\ln\left(\dfrac{\varepsilon-1}{\varepsilon c-1}\right)$
2 *Shell and tube:* One-shell pass 2, 4,…tube passes	$NTU = -\dfrac{1}{\sqrt{1+c^2}}\ln\left(\dfrac{2/\varepsilon-1-c-\sqrt{1+c^2}}{2/\varepsilon-1-c+\sqrt{1+c^2}}\right)$
3 *Cross-flow (single-pass):* C_{max} mixed, C_{min} unmixed	$NTU = -\ln\left[1+\dfrac{\ln(1-\varepsilon c)}{c}\right]$
C_{min} mixed, C_{max} unmixed	$NTU = -\dfrac{\ln[c\ln(1-\varepsilon)+1]}{c}$
4 *All heat exchangers with $c = 0$*	$NTU = -\ln(1-\varepsilon)$

When all the inlet and outlet temperatures are specified, the *size* of the heat exchanger can easily be determined using the LMTD method. Alternatively, it can be determined from the effectiveness–NTU method by first evaluating the effectiveness from its definition and then the NTU from the appropriate NTU relation.

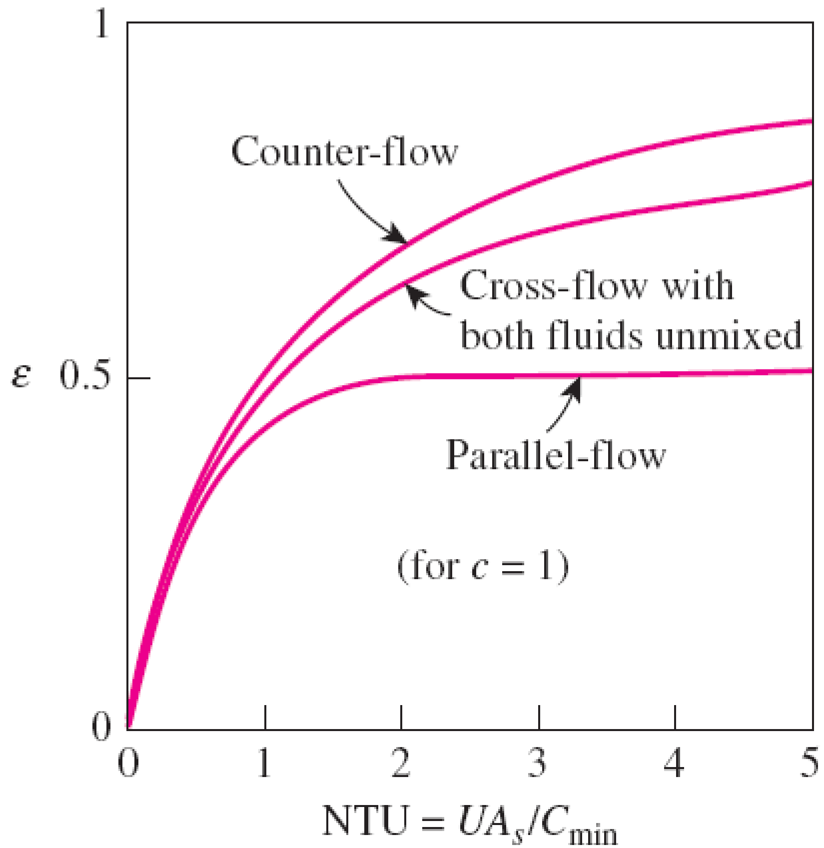


FIGURE 11–27

For a specified NTU and capacity ratio c, the counter-flow heat exchanger has the highest effectiveness and the parallel-flow the lowest.

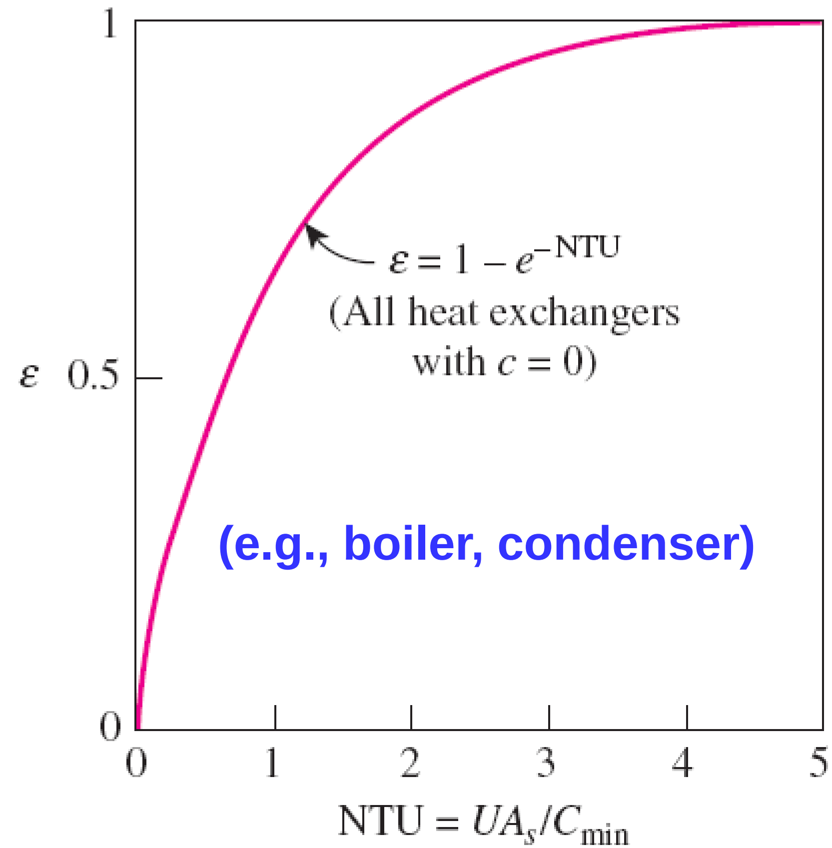


FIGURE 11–28

The effectiveness relation reduces to $\varepsilon = \varepsilon_{max} = 1 - \exp(-\text{NTU})$ for all heat exchangers when the capacity ratio $c = 0$.

Observations from the effectiveness relations and charts

- The value of the effectiveness ranges from 0 to 1. It increases rapidly with NTU for small values (up to about NTU = 1.5) but rather slowly for larger values. Therefore, the use of a heat exchanger with a large NTU (usually larger than 3) and thus a large size cannot be justified economically, since a large increase in NTU in this case corresponds to a small increase in effectiveness.
- For a given NTU and capacity ratio $c = C_{min} / C_{max}$, the *counter-flow* heat exchanger has the *highest* effectiveness, followed closely by the cross-flow heat exchangers with both fluids unmixed. The lowest effectiveness values are encountered in parallel-flow heat exchangers.
- The effectiveness of a heat exchanger is independent of the capacity ratio c for NTU values of less than about 0.3.
- The value of the capacity ratio c ranges between 0 and 1. For a given NTU, the effectiveness becomes a *maximum* for $c = 0$ (e.g., boiler, condenser) and a *minimum* for $c = 1$ (when the heat capacity rates of the two fluids are equal).

EXAMPLE 13–8 **Using the Effectiveness–NTU Method**

Repeat Example 13–4, which was solved with the LMTD method, using the effectiveness–NTU method.

EXAMPLE 13–4 **Heating Water in a Counter-Flow Heat Exchanger**

A counter-flow double-pipe heat exchanger is to heat water from 20°C to 80°C at a rate of 1.2 kg/s. The heating is to be accomplished by geothermal water available at 160°C at a mass flow rate of 2 kg/s. The inner tube is thin-walled and has a diameter of 1.5 cm. If the overall heat transfer coefficient of the heat exchanger is 640 $W/m^2 \cdot °C$, determine the length of the heat exchanger required to achieve the desired heating.

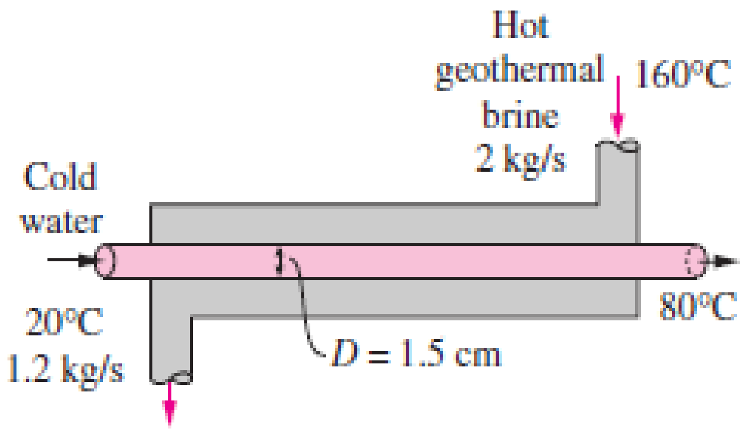


FIGURE 13–29
Schematic for Example 13–8.

SOLUTION The schematic of the heat exchanger is redrawn in Figure 13–29, and the same assumptions are utilized.

Analysis In the effectiveness–NTU method, we first determine the heat capacity rates of the hot and cold fluids and identify the smaller one:

$$C_h = \dot{m}_h C_{ph} = (2 \text{ kg/s})(4.31 \text{ kJ/kg} \cdot °\text{C}) = 8.62 \text{ kW/°C}$$
$$C_c = \dot{m}_c C_{pc} = (1.2 \text{ kg/s})(4.18 \text{ kJ/kg} \cdot °\text{C}) = 5.02 \text{ kW/°C}$$

Therefore,

$$C_{\min} = C_c = 5.02 \text{ kW/°C}$$

and

$$c = C_{\min}/C_{\max} = 5.02/8.62 = 0.583$$

Then the maximum heat transfer rate is determined from Eq. 13-32 to be

$$\begin{aligned}\dot{Q}_{\max} &= C_{\min}(T_{h,\text{ in}} - T_{c,\text{ in}}) \\ &= (5.02 \text{ kW/°C})(160 - 20)°\text{C} \\ &= 702.8 \text{ kW}\end{aligned}$$

That is, the maximum possible heat transfer rate in this heat exchanger is 702.8 kW. The actual rate of heat transfer in the heat exchanger is

$$\dot{Q} = [\dot{m}C_p(T_{\text{out}} - T_{\text{in}})]_{\text{water}} = (1.2 \text{ kg/s})(4.18 \text{ kJ/kg} \cdot °\text{C})(80 - 20)°\text{C} = 301.0 \text{ kW}$$

Thus, the effectiveness of the heat exchanger is

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{301.0 \text{ kW}}{702.8 \text{ kW}} = 0.428$$

Knowing the effectiveness, the NTU of this counter-flow heat exchanger can be determined from Figure 13–26*b* or the appropriate relation from Table 13–5. We choose the latter approach for greater accuracy:

$$\text{NTU} = \frac{1}{c-1}\ln\left(\frac{\varepsilon - 1}{\varepsilon c - 1}\right) = \frac{1}{0.583 - 1}\ln\left(\frac{0.428 - 1}{0.428 \times 0.583 - 1}\right) = 0.651$$

Then the heat transfer surface area becomes

$$\text{NTU} = \frac{UA_s}{C_{\min}} \longrightarrow A_s = \frac{\text{NTU}\ C_{\min}}{U} = \frac{(0.651)(5020\ \text{W/°C})}{640\ \text{W/m}^2 \cdot \text{°C}} = 5.11\ \text{m}^2$$

To provide this much heat transfer surface area, the length of the tube must be

$$A_s = \pi DL \longrightarrow L = \frac{A_s}{\pi D} = \frac{5.11\ \text{m}^2}{\pi(0.015\ \text{m})} = 108\ \text{m}$$

EXAMPLE 13–9 **Cooling Hot Oil by Water in a Multipass Heat Exchanger**

Hot oil is to be cooled by water in a 1-shell-pass and 8-tube-passes heat exchanger. The tubes are thin-walled and are made of copper with an internal diameter of 1.4 cm. The length of each tube pass in the heat exchanger is 5 m, and the overall heat transfer coefficient is 310 W/m^2 · °C. Water flows through the tubes at a rate of 0.2 kg/s, and the oil through the shell at a rate of 0.3 kg/s. The water and the oil enter at temperatures of 20°C and 150°C, respectively. Determine the rate of heat transfer in the heat exchanger and the outlet temperatures of the water and the oil.

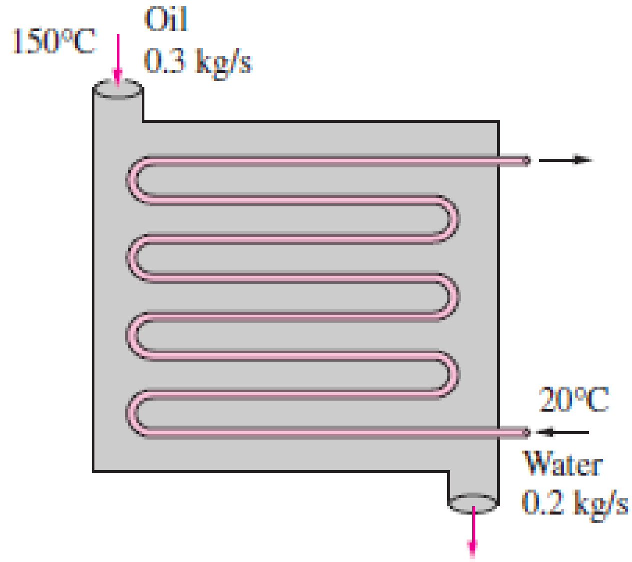


FIGURE 13–30
Schematic for Example 13–9.

SOLUTION Hot oil is to be cooled by water in a heat exchanger. The mass flow rates and the inlet temperatures are given. The rate of heat transfer and the outlet temperatures are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** The thickness of the tube is negligible since it is thin-walled. **4** Changes in the kinetic and potential energies of fluid streams are negligible. **5** The overall heat transfer coefficient is constant and uniform.

Analysis The schematic of the heat exchanger is given in Figure 13–30. The outlet temperatures are not specified, and they cannot be determined from an energy balance. The use of the LMTD method in this case will involve tedious iterations, and thus the ε–NTU method is indicated. The first step in the ε–NTU method is to determine the heat capacity rates of the hot and cold fluids and identify the smaller one:

$$C_h = \dot{m}_h C_{ph} = (0.3 \text{ kg/s})(2.13 \text{ kJ/kg} \cdot {}^\circ\text{C}) = 0.639 \text{ kW/}^\circ\text{C}$$
$$C_c = \dot{m}_c C_{pc} = (0.2 \text{ kg/s})(4.18 \text{ kJ/kg} \cdot {}^\circ\text{C}) = 0.836 \text{ kW/}^\circ\text{C}$$

Therefore,

$$C_{\min} = C_h = 0.639 \text{ kW/}^\circ\text{C}$$

and

$$c = \frac{C_{\min}}{C_{\max}} = \frac{0.639}{0.836} = 0.764$$

Then the maximum heat transfer rate is determined from Eq. 13-32 to be

$$\begin{aligned}\dot{Q}_{\max} &= C_{\min}(T_{h,\text{ in}} - T_{c,\text{ in}}) \\ &= (0.639 \text{ kW/}^\circ\text{C})(150 - 20)^\circ\text{C} = 83.1 \text{ kW}\end{aligned}$$

That is, the maximum possible heat transfer rate in this heat exchanger is 83.1 kW. The heat transfer surface area is kW. The heat transfer surface area is

$$A_s = n(\pi DL) = 8\pi(0.014 \text{ m})(5 \text{ m}) = 1.76 \text{ m}^2$$

Then the NTU of this heat exchanger becomes

$$\text{NTU} = \frac{UA_s}{C_{\min}} = \frac{(310 \text{ W/m}^2 \cdot {}^\circ\text{C})(1.76 \text{ m}^2)}{639 \text{ W/}^\circ\text{C}} = 0.853$$

The effectiveness of this heat exchanger corresponding to $c = 0.764$ and NTU = 0.853 is determined from Figure 13–26*c* to be

$$\varepsilon = 0.47$$

We could also determine the effectiveness from the third relation in Table 13–4 more accurately but with more labor. Then the actual rate of heat transfer becomes

$$\dot{Q} = \varepsilon \dot{Q}_{\max} = (0.47)(83.1 \text{ kW}) = \mathbf{39.1 \text{ kW}}$$

Finally, the outlet temperatures of the cold and the hot fluid streams are determined to be

$$\dot{Q} = C_c(T_{c,\text{ out}} - T_{c,\text{ in}}) \longrightarrow T_{c,\text{ out}} = T_{c,\text{ in}} + \frac{\dot{Q}}{C_c}$$

$$= 20^\circ\text{C} + \frac{39.1 \text{ kW}}{0.836 \text{ kW/}^\circ\text{C}} = \mathbf{66.8^\circ C}$$

$$\dot{Q} = C_h(T_{h,\text{ in}} - T_{h,\text{ out}}) \longrightarrow T_{h,\text{ out}} = T_{h,\text{ in}} - \frac{\dot{Q}}{C_h}$$

$$= 150^\circ\text{C} - \frac{39.1 \text{ kW}}{0.639 \text{ kW/}^\circ\text{C}} = \mathbf{88.8^\circ C}$$

Therefore, the temperature of the cooling water will rise from 20°C to 66.8°C as it cools the hot oil from 150°C to 88.8°C in this heat exchanger.

SELECTION OF HEAT EXCHANGERS

The uncertainty in the predicted value of *U* can exceed 30 percent. Thus, it is natural to tend to overdesign the heat exchangers.

Heat transfer enhancement in heat exchangers is usually accompanied by *increased pressure drop*, and thus *higher pumping power.*

Therefore, any gain from the enhancement in heat transfer should be weighed against the cost of the accompanying pressure drop.

Usually, the *more viscous fluid is more suitable for the shell side* (larger passage area and thus lower pressure drop) and *the fluid with the higher pressure for the tube side.*

The proper selection of a heat exchanger depends on several factors:

- **Heat Transfer Rate**
- **Cost**
- **Pumping Power**
- **Size and Weight**
- **Type**
- **Materials**

The *rate of heat transfer* in the prospective heat exchanger

$$\dot{Q} = \dot{m} c_p (T_{\text{in}} - T_{\text{out}})$$

The annual cost of electricity associated with the operation of the pumps and fans

$$\begin{aligned}\text{Operating cost} = &\ (\text{Pumping power, kW}) \times (\text{Hours of operation, h}) \\ &\times (\text{Unit cost of electricity, \$/kWh})\end{aligned}$$

EXAMPLE 13–10 **Installing a Heat Exchanger to Save Energy and Money**

In a dairy plant, milk is pasteurized by hot water supplied by a natural gas furnace. The hot water is then discharged to an open floor drain at 80°C at a rate of 15 kg/min. The plant operates 24 h a day and 365 days a year. The furnace has an efficiency of 80 percent, and the cost of the natural gas is $0.40 per therm (1 therm = 105,500 kJ). The average temperature of the cold water entering the furnace throughout the year is 15°C. The drained hot water cannot be returned to the furnace and recirculated, because it is contaminated during the process.

In order to save energy, installation of a water-to-water heat exchanger to preheat the incoming cold water by the drained hot water is proposed. Assuming that the heat exchanger will recover 75 percent of the available heat in the hot water, determine the heat transfer rating of the heat exchanger that needs to be purchased and suggest a suitable type. Also, determine the amount of money this heat exchanger will save the company per year from natural gas savings.

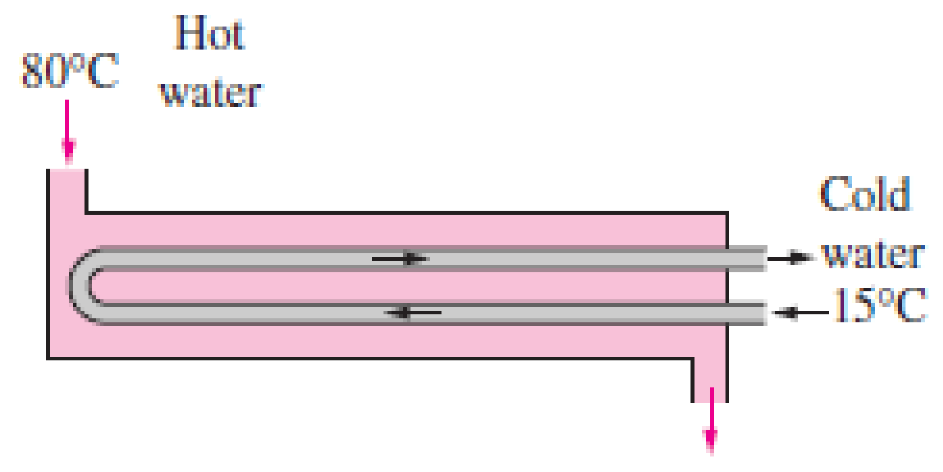


FIGURE 13–31
Schematic for Example 13–10.

SOLUTION A water-to-water heat exchanger is to be installed to transfer energy from drained hot water to the incoming cold water to preheat it. The rate of heat transfer in the heat exchanger and the amount of energy and money saved per year are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The effectiveness of the heat exchanger remains constant.

Properties We use the specific heat of water at room temperature, C_p = 4.18 kJ/kg · °C (Table A–9), and treat it as a constant.

Analysis A schematic of the prospective heat exchanger is given in Figure 13–31. The heat recovery from the hot water will be a maximum when it leaves the heat exchanger at the inlet temperature of the cold water. Therefore,

$$\begin{aligned}\dot{Q}_{\max} &= \dot{m}_h C_p (T_{h,\,\text{in}} - T_{c,\,\text{in}}) \\ &= \left(\frac{15}{60}\,\text{kg/s}\right)(4.18\ \text{kJ/kg}\cdot{}^\circ\text{C})(80 - 15)^\circ\text{C} \\ &= 67.9\ \text{kJ/s}\end{aligned}$$

That is, the existing hot water stream has the potential to supply heat at a rate of 67.9 kJ/s to the incoming cold water. This value would be approached in a counter-flow heat exchanger with a *very large* heat transfer surface area. A heat exchanger of reasonable size and cost can capture 75 percent of this heat transfer potential. Thus, the heat transfer rating of the prospective heat exchanger must be

$$\dot{Q} = \varepsilon \dot{Q}_{\max} = (0.75)(67.9\ \text{kJ/s}) = \mathbf{50.9\ kJ/s}$$

That is, the heat exchanger should be able to deliver heat at a rate of 50.9 kJ/s from the hot to the cold water. An ordinary plate or *shell-and-tube* heat exchanger should be adequate for this purpose, since both sides of the heat exchanger involve the same fluid at comparable flow rates and thus comparable heat transfer coefficients. (Note that if we were heating air with hot water, we would have to specify a heat exchanger that has a large surface area on the air side.)

The heat exchanger will operate 24 h a day and 365 days a year. Therefore, the annual operating hours are

$$\text{Operating hours} = (24\ \text{h/day})(365\ \text{days/year}) = 8760\ \text{h/year}$$

Noting that this heat exchanger saves 50.9 kJ of energy per second, the energy saved during an entire year will be

$$\begin{aligned}\text{Energy saved} &= (\text{Heat transfer rate})(\text{Operation time}) \\ &= (50.9\ \text{kJ/s})(8760\ \text{h/year})(3600\ \text{s/h}) \\ &= 1.605 \times 10^9\ \text{kJ/year}\end{aligned}$$

The furnace is said to be 80 percent efficient. That is, for each 80 units of heat supplied by the furnace, natural gas with an energy content of 100 units must be supplied to the furnace. Therefore, the energy savings determined above result in fuel savings in the amount of

$$\begin{aligned}\text{Fuel saved} &= \frac{\text{Energy saved}}{\text{Furnace efficiency}} = \frac{1.605 \times 10^9\ \text{kJ/year}}{0.80}\left(\frac{1\ \text{therm}}{105{,}500\ \text{kJ}}\right) \\ &= 19{,}020\ \text{therms/year}\end{aligned}$$

Noting that the price of natural gas is \$0.40 per therm, the amount of money saved becomes

$$\begin{aligned}\text{Money saved} &= (\text{Fuel saved}) \times (\text{Price of fuel}) \\ &= (19{,}020\ \text{therms/year})(\$0.40/\text{therm}) \\ &= \$7607/\text{year}\end{aligned}$$

Therefore, the installation of the proposed heat exchanger will save the company $7607 a year, and the installation cost of the heat exchanger will probably be paid from the fuel savings in a short time.

Summary

- Types of Heat Exchangers
- The Overall Heat Transfer Coefficient
  - ✓ Fouling factor
- Analysis of Heat Exchangers
- The Log Mean Temperature Difference Method
  - ✓ Counter-Flow Heat Exchangers
  - ✓ Multipass and Cross-Flow Heat Exchangers: Use of a Correction Factor
- The Effectiveness–NTU Method
- Selection of Heat Exchangers